§2-3 RATIONAL EXPRESSIONS

Definition

A **rational expression** has the form $\frac{P}{Q}$, where P and Q are polynomials and $Q \neq 0$.

Property

If $\frac{P}{Q}$ is a rational expression and if R is a polynomial such that $R \neq 0$, then $\frac{PR}{QR} = \frac{P}{Q}$.

Example 1 Simplify the rational expression: $\frac{2x-10}{2x^2-20x+50}$.

Solution

$$\begin{aligned}\frac{2x-10}{2x^2-20x+50} &= \frac{2(x-5)}{2(x^2-10x+25)} \\ &= \frac{2(x-5)}{2(x-5)^2} \\ &= \frac{1}{x-5}\end{aligned}$$

Procedure

Multiplying Rational Expressions

The product of the rational expressions $\frac{P}{Q}$ and $\frac{R}{S}$ is $\frac{P}{Q}\cdot\frac{R}{S} = \frac{PR}{QS}$.

Example 2 Multiply the following rational expressions: $\left(1+\frac{1}{x}\right)\left(\frac{3}{x+1}\right)$. Simplify your answer.

Solution

$$\begin{aligned}\left(1+\frac{1}{x}\right)\left(\frac{3}{x+1}\right) &= \left(\frac{x}{x}+\frac{1}{x}\right)\left(\frac{3}{x+1}\right) \\ &= \left(\frac{x+1}{x}\right)\left(\frac{3}{x+1}\right) \\ &= \frac{3(x+1)}{x(x+1)} \\ &= \frac{3}{x}\end{aligned}$$

Procedure

Dividing Rational Expressions

The quotient of the rational expressions $\frac{P}{Q}$ and $\frac{R}{S}$ is $\frac{P}{Q}\div\frac{R}{S} = \frac{P}{Q}\cdot\frac{S}{R} = \frac{PS}{QR}$.

Example 3 Divide the following rational expressions $\frac{x^2-x}{3}\div\frac{1}{3x}$. Simplify your answer.

Solution

$$\begin{aligned}\frac{x^2-x}{3}\div\frac{1}{3x} &= \frac{x(x-1)}{3}\cdot\frac{3x}{1} \\ &= \frac{3x^2(x-1)}{3} \\ &= x^2(x-1)\end{aligned}$$

Procedure

Adding Rational Expressions

If $\frac{P}{Q}$ and $\frac{R}{Q}$ are rational expressions, then $\frac{P}{Q}+\frac{R}{Q}=\frac{P+R}{Q}$.

Example 4 Add the following rational expressions: $\frac{2x}{x^2-1}+\frac{-1}{x-1}$. Simplify your answer.

Solution

$$\begin{aligned}\frac{2x}{x^2-1}+\frac{-1}{x-1} &= \frac{2x}{(x-1)(x+1)}+\frac{-1}{x-1}\\ &= \frac{2x}{(x-1)(x+1)}+\frac{-1}{(x-1)}\cdot\frac{(x+1)}{(x+1)}\\ &= \frac{2x-(x+1)}{(x-1)(x+1)}\\ &= \frac{2x-x-1}{(x-1)(x+1)}\\ &= \frac{x-1}{(x-1)(x+1)}\\ &= \frac{1}{x+1}\end{aligned}$$

Procedure

Subtracting Rational Expressions

If $\frac{P}{Q}$ and $\frac{R}{Q}$ are rational expressions, then $\frac{P}{Q}-\frac{R}{Q}=\frac{P-R}{Q}$.

Example 5 Subtract the following rational expressions: $\frac{1}{x^2-x-6}-\frac{2}{x^2+4x-21}$.

Solution

$$\begin{aligned}\frac{1}{x^2-x-6}-\frac{2}{x^2+4x-21} &= \frac{1}{(x-3)(x+2)}-\frac{2}{(x-3)(x+7)}\\ &= \frac{1\cdot(x+7)}{(x-3)(x+2)(x+7)}-\frac{2\cdot(x+2)}{(x-3)(x+7)(x+2)}\\ &= \frac{(x+7)-(2x+4)}{(x-3)(x+2)(x+7)}\\ &= \frac{x+7-2x-4}{(x-3)(x+2)(x+7)}\\ &= \frac{-(x-3)}{(x-3)(x+2)(x+7)}\\ &= \frac{-1}{(x+2)(x+7)}\\ &= \frac{-1}{x^2+9x+14}\end{aligned}$$

§2-3 PROBLEM SET

Simplify each expression.

1. $\frac{18xy}{12x}$ **2.** $\frac{(a+3)(a-2)}{(a+2)(a+3)}$ **3.** $\frac{3x-6}{9x}$

4. $\frac{3x^2+2x}{x}$ **5.** $\frac{h^2-16}{h+4}$ **6.** $\frac{x^2+2x}{x+2}$

7. $\frac{c^2+2c+1}{c^2-1}$ **8.** $\frac{z^2+4z}{z^2-2z}$ **9.** $\frac{(2t-1)(t+2)}{4t^2-4t+1}$

10. $\frac{x^2+6x+9}{x^3+27}$ **11.** $\frac{9x^2-4}{27x^3-8}$ **12.** $\frac{9m^2-16}{27m^3-64}$

Perform the indicated operation and simplify the result.

13. $\frac{a}{y}\cdot\frac{x}{a}$ **14.** $\frac{r}{s}\div\frac{t}{s}$ **15.** $\frac{2x}{3y}\cdot\frac{y^2}{4x}$

16. $\frac{14b}{9a}\div\frac{7a}{3}$ **17.** $\frac{12(t-1)}{t^2+1}\div\frac{t^2-1}{t+1}$ **18.** $\frac{3x^2+2x}{x-2}\cdot\frac{x-2}{x}$

19. $\frac{6y-2}{y}\cdot\frac{6y}{6y+3}$ **20.** $\frac{t+2}{t^2+3t}\div\frac{t+2}{t+3}$ **21.** $\frac{4k-8}{k+1}\div\frac{2k-10}{k+1}$

22. $\frac{5x^2+2x}{x-2}\div\frac{5x+2}{4x-8}$ **23.** $\frac{a^2+ab}{b}\cdot\frac{b}{a^2-ab}$ **24.** $\frac{10}{x+2}\cdot\frac{x^2+2x}{4x+6}$

25. $\frac{9x^2-4}{27x^3-8}\div\frac{3x+2}{3x-2}$ **26.** $\frac{x^2-x}{4x^2-9}\div\frac{x-1}{2x+3}$ **27.** $\frac{x^2+10x+25}{x-4}\cdot\frac{3x-12}{2x+10}$

28. $\frac{3}{a}+\frac{2}{b}$ **29.** $\frac{5}{x+2}+\frac{x}{x+2}$ **30.** $\frac{9}{2x+1}-\frac{5}{2x+1}$

31. $\frac{9}{r}+\frac{8}{r}$ **32.** $\frac{x}{z}+\frac{z}{y}$ **33.** $\frac{5}{xy}-3$

34. $\frac{8}{m+3}+\frac{2}{m}$ **35.** $\frac{b}{b-2}+\frac{4}{b-3}$ **36.** $\frac{n}{n+3}+\frac{3}{n+3}$

37. $\frac{z}{z+2}-\frac{4}{z-2}$ **38.** $\frac{10}{x-y}-\frac{2}{y-x}$ **39.** $\frac{3}{x}+\frac{2}{x-2}$

40. $\frac{t}{t+2}+\frac{2}{t-1}$ **41.** $\frac{2x}{x-1}-\frac{5}{x+1}$ **42.** $\frac{y+2}{y^2-4}+\frac{y^2+2y+4}{y^3-8}$

§2-3 PROBLEM SOLUTIONS

1. $\frac{3y}{2}$ **2.** $\frac{a-2}{a+2}$ **3.** $\frac{x-2}{3x}$

4. $3x + 2$ **5.** $h - 4$ **6.** x

7. $\frac{c+1}{c-1}$ **8.** $\frac{z+4}{z-2}$ **9.** $\frac{t+2}{2t-1}$

10. $\frac{x+3}{x^2+3x+9}$ **11.** $\frac{3x+2}{9x^2-6x+4}$ **12.** $\frac{3m+4}{9m^2-12m+16}$

13. $\frac{x}{y}$ **14.** $\frac{r}{t}$ **15.** $\frac{y}{6}$

16. $\frac{2b}{3a^2}$ **17.** $\frac{12}{t^2+1}$ **18.** $3x + 2$

19. $\frac{12y-4}{2y+1}$ **20.** $\frac{1}{t}$ **21.** $\frac{2k-4}{k-5}$

22. $4x$ **23.** $\frac{a+b}{a-b}$ **24.** $\frac{5x}{2x+3}$

25. $\frac{3x-2}{9x^2-6x+4}$ **26.** $\frac{x}{2x-3}$ **27.** $\frac{3x+15}{2}$

28. $\frac{2a+3b}{ab}$ **29.** $\frac{x+5}{x+2}$ **30.** $\frac{4}{2x+1}$

31. $\frac{17}{r}$ **32.** $\frac{z^2+xy}{yz}$ **33.** $\frac{5-3xy}{xy}$

34. $\frac{10m+6}{m^2+3m}$ **35.** $\frac{b^2+b-8}{b^2-5b+6}$ **36.** 1

37. $\frac{z-4}{z-2}$ **38.** $\frac{12}{x-y}$ **39.** $\frac{5x-6}{x^2-2x}$

40. $\frac{t^2+t+4}{t^2+t-2}$ **41.** $\frac{2x^2-3x+5}{x^2-1}$ **42.** $\frac{2}{y-2}$