

**§2-4****RADICAL EXPRESSIONS****Definition**

The  **$n$ th root** of a number  $a$  is a number that when raised to the  $n$ th power, produces the number  $a$ . It is written as  $\sqrt[n]{a}$ , where  $n$  is the **index**,  $\sqrt{\quad}$  is the **radical** and  $a$  is the **radicand**.

For example,  $\sqrt{9} = 3$  since  $3^2 = 9$  and  $\sqrt{x^6} = x^3$  since  $(x^3)^2 = x^6$ .  
Similarly,  $\sqrt[4]{16} = 2$  since  $2^4 = 16$  and  $\sqrt[3]{x^{12}} = x^4$  since  $(x^4)^3 = x^{12}$ .

**Definition**

**Rational exponents** are exponents of the form  $\frac{m}{n}$ , where  $m$  is the power of the number and  $n$  represents the  $n$ th root of the number. Thus  $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$  where  $a \geq 0$ .

For example,  $27^{1/3} = \sqrt[3]{27} = 3$  and  $8^{2/3} = (\sqrt[3]{8})^2 = (2)^2 = 4$

**Example 1** Write the expression  $x^{2/3}$  in radical form.

**Solution** The expression  $x^{2/3}$  can be written as  $(x^{1/3})^2$  or  $(x^2)^{1/3}$ .  
Therefore,  $x^{2/3}$  can be written in radical form as  $(\sqrt[3]{x})^2$  or  $\sqrt[3]{x^2}$ .

**Example 2** Simplify the expression  $\sqrt{18} - \sqrt{8}$ .

**Solution**  $\sqrt{18} - \sqrt{8} = \sqrt{9 \cdot 2} - \sqrt{4 \cdot 2} = 3\sqrt{2} - 2\sqrt{2} = (3 - 2)\sqrt{2} = \sqrt{2}$

**Example 3** Simplify the expression  $\sqrt[3]{x^7 y^5 z^6} - \sqrt[3]{x^{10} y^8 z^3}$ .

**Solution**

$$\begin{aligned}\sqrt[3]{x^7 y^5 z^6} - \sqrt[3]{x^{10} y^8 z^3} &= \sqrt[3]{x^6 \cdot x \cdot y^3 \cdot y^2 \cdot z^6} - \sqrt[3]{x^9 \cdot x \cdot y^6 \cdot y^2 \cdot z^3} \\ &= (x^2 y z^2) \sqrt[3]{x y^2} - (x^3 y^2 z) \sqrt[3]{x y^2} \\ &= (x^2 y z^2 - x^3 y^2 z) \sqrt[3]{x y^2} \\ &= x^2 y z (z - xy) \sqrt[3]{x y^2}\end{aligned}$$

**Example 4** Multiply  $(\sqrt{2} - \sqrt{3})(\sqrt{8} - \sqrt{4})$ .

**Solution**

$$\begin{aligned}(\sqrt{2} - \sqrt{3})(\sqrt{8} - \sqrt{4}) &= \sqrt{2} \cdot \sqrt{8} - \sqrt{2} \cdot \sqrt{4} - \sqrt{3} \cdot \sqrt{8} + \sqrt{3} \cdot \sqrt{4} \\ &= \sqrt{16} - \sqrt{8} - \sqrt{24} + \sqrt{12} \\ &= 4 - \sqrt{4 \cdot 2} - \sqrt{4 \cdot 6} + \sqrt{4 \cdot 3} \\ &= 4 - 2\sqrt{2} - 2\sqrt{6} + 2\sqrt{3}\end{aligned}$$

Write each exponential expression as a radical expression.

1.  $5^{1/2}$       2.  $(2x)^{1/3}$       3.  $(xy^3)^{3/4}$       4.  $-5x^{2/3}$       5.  $(a^2b^4)^{1/5}$

Write each radical expression as a exponential expression.

6.  $\sqrt{11}$       7.  $\sqrt[5]{a^3}$       8.  $-\sqrt{2y}$       9.  $4x\sqrt{x}$       10.  $2r^4\sqrt[4]{2s^3}$

Simplify each radical expression.

11.  $\sqrt{64}$       12.  $\sqrt[3]{64}$       13.  $\sqrt[3]{-1}$       14.  $\sqrt[3]{-27}$       15.  $\sqrt{\frac{4}{9}}$

16.  $\sqrt{x^{12}}$       17.  $-\sqrt{a^{12}b^4}$       18.  $\sqrt[3]{8x^6y^9}$       19.  $\sqrt[4]{16r^8}$       20.  $\sqrt[5]{-x^5y^{10}}$

21.  $\sqrt[3]{-27a^9}$       22.  $\sqrt[5]{243x^{15}y^{25}}$       23.  $\sqrt{\frac{25x^2}{y^4}}$       24.  $\sqrt[3]{\frac{27a^{12}}{b^{15}}}$

25.  $\sqrt{(x+y)^2}$       26.  $\sqrt[3]{(x+y)^6}$       27.  $3\sqrt{5} + 2\sqrt{5}$       28.  $3\sqrt{5} - 2\sqrt{5}$

29.  $2\sqrt{12} - 3\sqrt{3}$       30.  $a\sqrt{7} - b\sqrt{7}$       31.  $3 \cdot \sqrt[3]{16} - \sqrt[3]{2}$       32.  $\sqrt{x^3} - 2x\sqrt{x}$

33.  $\sqrt{x} + \sqrt{x}$       34.  $\sqrt{x} \cdot \sqrt{x}$       35.  $3\sqrt{2} \cdot \sqrt{3}$       36.  $\sqrt{x+y} \cdot \sqrt{x+y}$

37.  $\sqrt{6x} \cdot \sqrt{12x}$       38.  $\frac{2}{\sqrt{2}}$       39.  $\frac{x}{\sqrt{x}}$       40.  $\frac{2x}{\sqrt{8x^3}}$

41.  $\sqrt[3]{\sqrt{64}}$       42.  $\sqrt[3]{\sqrt{x}}$       43.  $\sqrt[3]{\sqrt{x^{12}}}$       44.  $\sqrt{\sqrt[3]{4ab^2}}$

45.  $\frac{5\sqrt{x}}{y^2} \cdot \frac{\sqrt{25y}}{\sqrt{x^3}}$       46.  $\frac{\sqrt{3y}}{4\sqrt{4x}} \cdot \frac{x}{y\sqrt{y}}$       47.  $\frac{5\sqrt{x}}{y^2} \div \frac{\sqrt{25y}}{\sqrt{x^3}}$       48.  $\frac{\sqrt{3y}}{4\sqrt{4x}} \div \frac{x}{y\sqrt{y}}$

49.  $\frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}}$       50.  $\frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}}$       51.  $\frac{\sqrt{27}}{\sqrt{12}} + \frac{2}{\sqrt[3]{27}}$

52.  $\frac{2}{\sqrt{12}} - \frac{3}{\sqrt{3}}$       53.  $\frac{2}{\sqrt{12}} + \frac{3}{\sqrt{3}}$       54.  $\frac{\sqrt{27}}{\sqrt{12}} - \frac{2}{\sqrt[3]{27}}$

55.  $\sqrt{2}(\sqrt{10} + \sqrt{5})$       56.  $\sqrt{3}(\sqrt{8} - \sqrt{6})$       57.  $\frac{2}{\sqrt{3} - \sqrt{2}}$

58.  $\frac{2}{\sqrt{3} + \sqrt{2}}$       59.  $\frac{3}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{3} - \sqrt{2}}$       60.  $\frac{2}{\sqrt{2} + \sqrt{5}} - \frac{3}{\sqrt{2} - \sqrt{5}}$

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|-----|-------------------------|-----|----------------------------------|-----|-----------------------------|-----|--------------------------|-----|---------------------------|
| 1.  | $\sqrt{5}$              | 2.  | $\sqrt[3]{2x}$                   | 3.  | $y^2 \cdot \sqrt[4]{x^3y}$  | 4.  | $-5 \cdot \sqrt[3]{x^2}$ | 5.  | $\sqrt[5]{a^2b^4}$        |
| 6.  | $11^{1/2}$              | 7.  | $a^{3/5}$                        | 8.  | $-(2y)^{1/2}$               | 9.  | $4x^{3/2}$               | 10. | $(2^{7/4})(r)(s^{3/4})$   |
| 11. | 8                       | 12. | 4                                | 13. | -1                          | 14. | -3                       | 15. | $\frac{2}{3}$             |
| 16. | $x^6$                   | 17. | $-a^6b^2$                        | 18. | $2x^2y^3$                   | 19. | $2r^2$                   | 20. | $-xy^2$                   |
| 21. | $-3a^3$                 | 22. | $3x^3y^5$                        | 23. | $\frac{5x}{y^2}$            | 24. | $\frac{3a^4}{b^5}$       | 25. | $x + y$                   |
| 26. | $(x + y)^2$             | 27. | $5\sqrt{5}$                      | 28. | $\sqrt{5}$                  | 29. | $\sqrt{3}$               | 30. | $(a - b)\sqrt{7}$         |
| 31. | $5 \cdot \sqrt[3]{2}$   | 32. | $-x\sqrt{x}$                     | 33. | $2\sqrt{x}$                 | 34. | $x$                      | 35. | $3\sqrt{6}$               |
| 36. | $x + y$                 | 37. | $2x\sqrt{18}$                    | 38. | $\sqrt{2}$                  | 39. | $\sqrt{x}$               | 40. | $\frac{\sqrt{2x}}{2x}$    |
| 41. | 2                       | 42. | $\sqrt[6]{x}$                    | 43. | $x^2$                       | 44. | $\sqrt[24]{ab^2}$        | 45. | $\frac{25\sqrt{y}}{xy^2}$ |
| 46. | $\frac{\sqrt{3x}}{8y}$  | 47. | $\frac{x^2\sqrt{y}}{y^3}$        | 48. | $\frac{y^2\sqrt{3x}}{8x^2}$ | 49. | $\frac{7\sqrt{2}}{2}$    | 50. | $\frac{\sqrt{2}}{2}$      |
| 51. | $\frac{13}{6}$          | 52. | $-\frac{2\sqrt{3}}{3}$           | 53. | $\frac{4\sqrt{3}}{3}$       | 54. | $\frac{5}{6}$            | 55. | $2\sqrt{5} + \sqrt{10}$   |
| 56. | $2\sqrt{6} - 3\sqrt{2}$ | 57. | $2\sqrt{3} + 2\sqrt{2}$          | 58. | $2\sqrt{3} - 2\sqrt{2}$     |     |                          |     |                           |
| 59. | $4\sqrt{3} - 2\sqrt{2}$ | 60. | $\frac{\sqrt{2} + 5\sqrt{5}}{3}$ |     |                             |     |                          |     |                           |