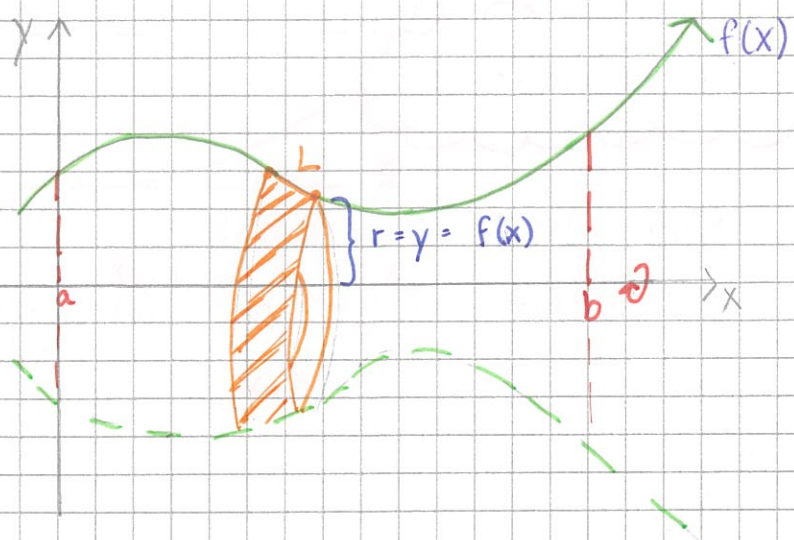
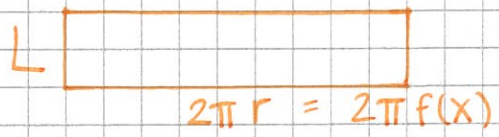
Section 5.5 Area of a Surface of Revolution





∴ For one arbitrary ring, the area of the ring is equal to:

Surface area → $A_{Ring} = 2\pi r \cdot L$

$= 2\pi f(x) \cdot L$

$$\therefore S = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}\,dx$$

Example 1: Find the area of the surface generated by revolving the portion of the curve $y = x^3$ on the interval $[0,1]$ about the x-axis.

$f(x) = x^3$

$f'(x) = 3x^2$

$$S = 2\pi \int_0^1 x^3 \sqrt{1+(3x^2)^2}\, dx$$

$$= 2\pi \int_0^1 x^3 \sqrt{1+9x^4}\, dx \quad \frac{1}{36}\, du$$

(x^3 and dx are grouped; $\sqrt{1+9x^4}$ is u)

$u = 1 + 9x$ $\qquad$ $x = 1 \rightarrow u = 10$

$du = 36x^3\, dx$ $\qquad$ $x = 0 \rightarrow u = 1$

$\frac{1}{36}\, du = x^3\, dx$

$$S = 2\pi \cdot \frac{1}{36} \int_1^{10} u^{1/2}\, du$$

$$= \frac{\pi}{18} \cdot \frac{2}{3} u^{3/2} \Big|_1^{10}$$

$$= \frac{\pi}{27} \left[u^{3/2} \right]_1^{10}$$

$$= \frac{\pi}{27} \left[10^{3/2} - 1^{3/2} \right]$$

$$= \frac{\pi}{27} \left[10\sqrt{10} - 1 \right]$$

$$= \frac{(10\sqrt{10} - 1)\pi}{27}$$