Exponential and Logarithmic Functions
Solutions To Selected Problems
Calculus 9^{th} Edition Anton, Bivens, Davis

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1. Simplify the following without using a calculator.

(a) $$-8^{2/3} = -\left(\sqrt[3]{8}\right)^2 = -(2)^2 = \boxed{-4}$$

(b) $$(-8)^{2/3} = \left(\sqrt[3]{-8}\right)^2 = (-2)^2 = \boxed{4}$$

(c) $$8^{-2/3} = \frac{1}{\left(\sqrt[3]{-8}\right)^2} = \boxed{\frac{1}{4}}$$

5. Find the exact value of the expression without a calculator.

(a) $$\log_2(16) = \log_2(2^4) = 4\log_2(2) = 4 \cdot (1) = \boxed{4}$$

(b) $$\log_2\left(\frac{1}{32}\right) = \log_2(32^{-1}) = \log_2((2^5)^{-1}) = \log_2(2^{-5}) = -5\log_2(2) = -5 \cdot (1) = \boxed{-5}$$

(c) $$\log_4(4) = \boxed{1}$$

(d) $$\log_9 3 = \log_9\left(9^{1/2}\right) = \frac{1}{2}\log_9(9) = \frac{1}{2} \cdot (1) = \boxed{\frac{1}{2}}$$

9. Use the logarithm properties in Theorem 6.1.3 to rewrite the expression in terms of $r,\ s$, and t, where $r = \ln(a),\ s = \ln(b),\ t = \ln(c)$

(a)
$$\begin{aligned}\ln\left(a^2\sqrt{bc}\,\right) &= \ln(a^2) + \ln\left(\sqrt{bc}\,\right)\\ &= 2\ln(a) + \frac{1}{2}\ln(bc) = 2\ln(a) + \frac{1}{2}\left(\ln(b) + \ln(c)\right) = \boxed{2r + \frac{1}{2}(s+t)}\end{aligned}$$

11. Expand the logarithm in terms of sums, differences, and multiples of simpler logarithms.

(b)
$$\begin{aligned}\ln\left(\frac{x^2\sin^3(x)}{\sqrt{x^2+1}}\right) &= \log(x^2) + \log(\sin^3(x)) - \log(x^2+1)^{1/2}\\ &= \boxed{2\log(x) + 3\log(\sin(x)) - \frac{1}{2}\log(x^2+1)}\end{aligned}$$

13.
$$4\log(2) - \log(3) + \log(16)$$

$$= \log(2^4) + \log(16) - \log(3)$$

$$= \log\left(\frac{2^4\cdot 16}{3}\right) = \log\left(\frac{16\cdot 16}{3}\right) = \boxed{\log\left(\frac{256}{3}\right)}$$

17.
$$\begin{aligned}\log_{10}(\sqrt{x}\,) &= -1\\ 10^{-1} &= \sqrt{x}\\ \left(10^{-1}\right)^2 &= (\sqrt{x}\,)^2\\ 10^{-2} &= x\\ \boxed{\frac{1}{100} = x}\end{aligned}$$

19.
$$\begin{aligned} \ln\left(\frac{1}{x}\right) &= -2 \\ e^{-2} &= x^{-1} \\ \frac{1}{e^2} &= \frac{1}{x} \\ \boxed{e^2 = x} \end{aligned}$$

21.
$$\begin{aligned} \log_5\left(5^{2x}\right) &= 8 \\ 5^8 &= 5^{2x} \\ 8 &= 2x \\ \boxed{4 = x} \end{aligned}$$

24.
$$\begin{aligned} 3^x &= 2 \\ \ln 3^x &= \ln 2 \\ x \ln 3 &= \ln 2 \\ \boxed{x = \frac{\ln 2}{\ln 3}} \end{aligned}$$

25.
$$\begin{aligned} 5^{-2x} &= 3 \\ -2x \ln 5 &= \ln 3 \\ -2x &= \frac{\ln 3}{\ln 5} \\ \boxed{x = -\frac{\ln 3}{2 \ln 5}} \end{aligned}$$

27.

$$2e^{3x} = 7$$

$$e^{3x} = \frac{7}{2}$$

$$\ln e^{3x} = \ln \frac{7}{2}$$

$$3x \ln e = \ln \frac{7}{2}$$

$$3x \cdot 1 = \ln \frac{7}{2}$$

$$\boxed{x = \frac{1}{3} \ln \frac{7}{2}}$$

29.

$$xe^{-x} + 2e^{-x} = 0$$
$$e^{-x}(x+2) = 0$$

$$e^{-x} = 0 \rightarrow DNE$$

$$x + 2 = 0$$
$$\boxed{x = -2}$$

30.

$$e^{-2x} - 3e^{-x} = -2$$

$$\left(e^{-x}\right)^2 - 3e^{-x} + 2 = 0$$

$$Let\ u = e^{-x}$$

$$\rightarrow u^2 - 3u + 2 = 0$$

$$(u-1)(u-2) = 0$$

$$u = 1\ or\ u = 2$$

$$u = 1 \rightarrow e^{-x} = 1$$

$$\ln e^{-x} = \ln 1$$

$$-x = 0$$

$$\boxed{x = 0}$$

$$u = 2 \rightarrow e^{-x} = 2$$

$$-x \ln e = \ln 2$$

$$\boxed{x = -\ln 2}$$