An Overview of the Area Problem
Solutions To Selected Problems
Calculus 9^{th} Edition Anton, Bivens, Davis

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1. Estimate the area between the graph of the function f and the interval $[a, b]$ using $n = 2,\ and\ 5$ rectangles.

   (a) $f(x) = \sqrt{x}; \quad [a, b] = [0, 1].$

$$
\begin{aligned}
\underline{n = 2} \quad &\rightarrow \frac{1-0}{2} = \frac{1}{2} \\
x &= 0,\ \frac{1}{2},\ 1 \\
A_2 &= [f(0) + f(1/2) + f(1)]\frac{1}{2} \\
&= \left[\sqrt{0} + \sqrt{1/2} + \sqrt{1}\right]\frac{1}{2} \\
&= \left[\frac{1}{\sqrt{2}} + 1\right]\frac{1}{2} \\
&= \frac{1}{2\sqrt{2}} + \frac{1}{2} \\
&= \frac{1+\sqrt{2}}{2\sqrt{2}} \approx \frac{2.414213}{2.828427} \approx \boxed{0.853553}
\end{aligned}
$$

$$
\begin{aligned}
\underline{n = 5} \quad &\rightarrow \frac{1-0}{5} = \frac{1}{5} \\
x &= 0,\ 1/5,\ 2/5.\ 3/5,\ 4/5,\ 1 \\
A_5 &= [f(0) + f(1/5) + f(2/5) + f(3/5) + f(4/5) + f(1)]\frac{1}{5} \\
&= \left[\sqrt{0} + \sqrt{1/5} + \sqrt{2/5} + \sqrt{3/5} + \sqrt{4/5} + \sqrt{1}\right] \\
&= \left[\frac{1+\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{5}}{5\sqrt{5}}\right] \\
&\approx \frac{8.3823323}{11.180339} \approx \boxed{0.74974}
\end{aligned}
$$

(b) $f(x) = \sin(x); \quad [a, b] = [0, \pi]$

$$\underline{n=2} \quad \to \frac{\pi - 0}{2} = \frac{\pi}{2}$$

$$x = 0,\ \pi/2,\ \pi$$

$$A_2 = [\sin(0) + \sin(\pi/2) + \sin(\pi)]\frac{\pi}{2}$$

$$= [0 + 1 + 0]\frac{\pi}{2} = \frac{\pi}{2} \approx \boxed{1.57080}$$

$$\underline{n=5} \quad \to \frac{\pi - 0}{5} = \frac{\pi}{5}$$

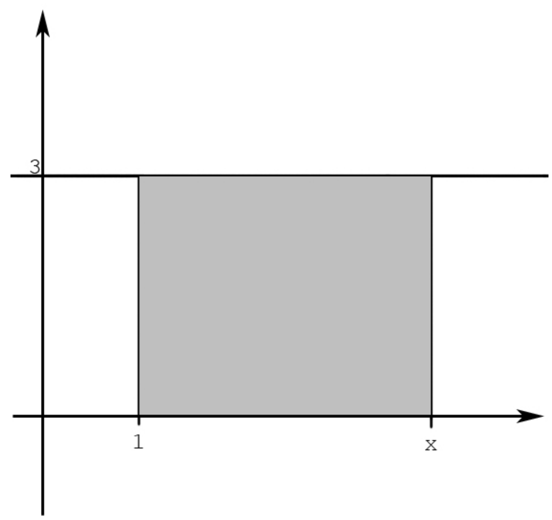
$$x = 0,\ \pi/5,\ 2\pi/5,\ 3\pi/5,\ 4\pi/5, \pi$$

$$A_5 = [\sin(0) + \sin(\pi/5) + \sin(2\pi/5) + \sin(3\pi/5) + \sin(4\pi/5) + \sin(\pi)]\frac{\pi}{5}$$

$$= [3.07768353](0.62831853) \approx \boxed{1.93376}$$

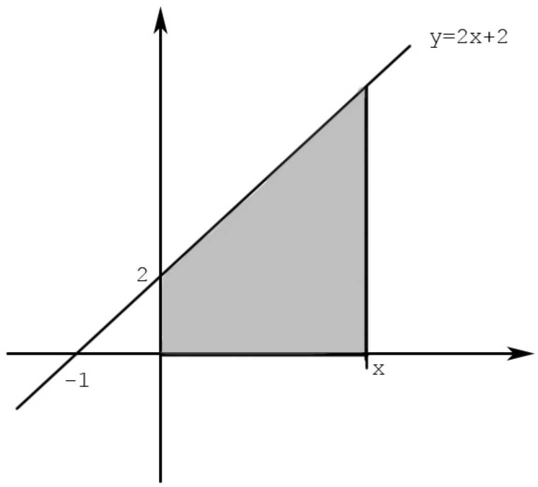
2. Graph each function over the specified interval. Then use simple area formulas from geometry to find the area function $A(x)$ that gives the area between the graph of the specified function f and the interval $[a, x]$. Confirm that $A'(x) = f(x)$

   (a) $f(x) = 3; \quad [a, x] = [1, x]$



This is just a rectangle with a height of 3 and a base of $x - 1$, So the area function $A(x) = 3(x - 1) = 3x - 3$, with $A'(x) = 3 = f(x)$,

(b) $f(x) = 2x + 2; \quad [a, x] = [0, x]$



We only want the area of the shaded part, so we need to find the area of the entire triangle and then subtract the area of the smaller triangle.

The height of the big triangle is $2x + 2$ and the base is $1 + x$. So its area is $\frac{1}{2}(2x + 2)(x + 1) = \frac{1}{2}(2x^2 + 4x + 2) = x^2 + 2x + 1$.

The smaller triangle has a base of 1 and a height of 2, so its area is $\frac{1}{2}(2)(1) = 1$.

So we have the difference of the two areas to give us $A(x) = x^2+2x+1-1 = x^2 + 2x = x(x + 2)$.

$A'(x) = 2x + 2$.