The Indefinite Integral
Solutions To Selected Problems
Calculus 9^{th} Edition Anton, Bivens, Davis

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November 15, 2011

1. Confirm that the formula is correct, and state a corresponding integration formula.

(a)
$$\frac{d}{dx}[\sqrt{1+x^2}] = \frac{x}{\sqrt{1+x^2}}$$

$$\begin{aligned}\frac{d}{dx}[\sqrt{1+x^2}] &= \frac{1}{2}(1+x^2)^{-1/2}(2x) \\ &= \frac{2x}{2\sqrt{1+x^2}} \\ &= \frac{x}{\sqrt{1+x^2}}\end{aligned}$$

$$\boxed{\int \frac{x}{\sqrt{1+x^2}}\ dx = \sqrt{1+x^2} + C}$$

(b)
$$\frac{d}{dx}\left[\frac{1}{3}\sin(1+x^3)\right] = x^2\cos(1+x^3)$$

$$\begin{aligned}\frac{d}{dx}\left[\frac{1}{3}\sin(1+x^3)\right] &= \frac{1}{3}\cos(1+x^3)(3x^2) \\ &= x^2\cos(1+x^3)\end{aligned}$$

$$\boxed{\int x^2\cos(1+x^3)\ dx = \frac{1}{3}\sin(1+x^3) + C}$$

2. Find the Derivative and state a corresponding integration formula.

(a)
$$\begin{aligned}\frac{d}{dx}[\sqrt{x^3+5}] &= \frac{1}{2}(x^3+5)^{-1/2}(3x^2) \\ &= \frac{3x^2}{2\sqrt{x^3+5}}\end{aligned}$$

$$\boxed{\int \frac{3x^2}{2\sqrt{x^3+5}}\ dx = \sqrt{x^3+5} + C}$$

(b)

$$\begin{aligned}\frac{d}{dx}[\sin(2\sqrt{x})] &= \cos(2\sqrt{x})\frac{d}{dx}(2\sqrt{x})\\ &= \cos(2\sqrt{x})(2\frac{1}{2\sqrt{x}})\\ &= \frac{\cos(2\sqrt{x})}{\sqrt{x}}\end{aligned}$$

$$\boxed{\int \frac{\cos(2\sqrt{x})}{\sqrt{x}}\ dx = \sin(2\sqrt{x}) + C}$$

3. Evaluate the integral by rewriting the integrand appropriately, if required, and applying the power rule.

(a)

$$\int x^8\ dx = \frac{x^{8+1}}{8+1} + C = \boxed{\frac{x^9}{9} + C}$$

(b)

$$\int x^{5/7}\ dx = \frac{x^{5/7+1}}{5/7+1} + C = \frac{x^{12/7}}{\frac{12}{7}} + C = \boxed{\frac{7}{12}x^{12/7} + C}$$

(c)

$$\begin{aligned}\int x^3\sqrt{x}\ dx &= \int x^3 x^{1/2}\ dx\\ &= \int x^{3+1/2}\ dx\\ &= \int x^{7/2}\ dx\\ &= \frac{x^{7/2+1}}{7/2+1} + C\\ &= \frac{x^{9/2}}{\frac{9}{2}} + C\\ &= \boxed{\frac{2}{9}x^{9/2} + C}\end{aligned}$$

4. Evaluate each integral by applying Theorem 4.2.4 and formula 2 in Table 4.2.1 appropriately,

(a)

$$\int \left[5x + \frac{2}{3x^5}\right] \, dx$$

$$= 5\int x \, dx + \frac{2}{3}\int x^{-5} \, dx$$

$$= 5\left[\frac{1}{2}x^2\right] + \frac{2}{3}\frac{x^{-4}}{-4} + C$$

$$= \boxed{\frac{5}{2}x^2 - \frac{1}{6x^4} + C}$$

(b)

$$\int \left[x^{-3} - 3x^{1/4} + 8x^2\right] \, dx$$

$$= \int x^{-3} \, dx - 3\int x^{1/4} \, dx + 8\int x^2 \, dx$$

$$= \frac{x^{-2}}{-2} - 3\frac{x^{1/4+1}}{1/4+1} + 8\frac{x^3}{3} + C$$

$$= -\frac{1}{2x^2} - 3\left(\frac{4}{5}\right)x^{5/4} + \frac{8}{3}x^3 + C$$

$$= \boxed{-\frac{1}{2x^2} + \frac{12}{5}x^{5/4} + \frac{8}{3}x^3 + C}$$

5. Evaluate each integral.

(a)

$$\int x(1+x^2)\ dx$$

$$= \int (x+x^4)\ dx$$

$$= \int x\ dx + \int x^4\ dx$$

$$= \boxed{\frac{x^2}{2} + \frac{x^5}{5} + C}$$

(b)

$$\int x^{1/3}(2-x)^2\ dx$$

$$= \int x^{1/3}(4-4x+x^2)\ dx$$

$$= \int 4x^{1/3} - 4xx^{1/3} + x^2x^{1/3}\ dx$$

$$= 4\int x^{1/3}\ dx - 4\int x^{4/3}\ dx + \int x^{7/3}\ dx$$

$$= 4\frac{x^{1/3+1}}{1/3+1} - 4\frac{x^{4/3}+1}{4/3+1} + \frac{x^{7/3+1}}{7/3+1} + C$$

$$= 4\frac{3}{4}x^{4/3} - 4\frac{3}{7}x^{7/3} + \frac{3}{10}x^{10/3} + C$$

$$= \boxed{3x^{4/3} - \frac{12}{7}x^{7/3} + \frac{3}{10}x^{10/3} + C}$$

(c)

$$\int \frac{x^5+2x^2-1}{x^4}\ dx$$

$$= \int \frac{x^5}{x^4} + \frac{2x^2}{x^4} - \frac{1}{x^4}\ dx = \int x\ dx + 2\int x^{-2}\ dx - \int x^{-4}\ dx$$

$$= \frac{x^2}{2} + \frac{2x^{-1}}{-1} - \frac{x^{-3}}{-3} + C$$

$$= \boxed{\frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + C}$$

(d)

$$\int (3\sin(x) - 2\sec^2(x))\ dx$$
$$= 3\int \sin(x)\ dx - 2\int \sec^2(x)\ dx$$
$$= \boxed{-3\cos(x) - 2\tan(x) + C}$$

(e)

$$\int \sec(x)(\sec(x) + \tan(x))\ dx$$
$$= \int (\sec^2(x) + \sec(x)\tan(x))\ dx$$
$$= \int \sec^2(x)\ dx + \int \sec(x)\tan(x)\ dx$$
$$= \boxed{\tan(x) + \sec(x) + C}$$

(f)

$$\int \frac{\sec(\theta)}{\cos(\theta)}\ d\theta$$
$$= \int \sec(\theta)\sec(\theta)\ d\theta$$
$$= \int \sec^2(\theta)\ d\theta = \boxed{\tan(\theta) + C}$$

(g)

$$\int \frac{\sin(x)}{\cos^2(x)}\ dx$$
$$= \int \frac{\sin(x)}{\cos(x)}\frac{1}{\cos(x)}\ dx$$
$$= \int \tan(x)\sec(x)\ dx = \boxed{\sec(x) + C}$$

(h)

$$\int [1 + \sin^2(y)\csc(y)]\ dy = \int 1 + \sin^2(y)\frac{1}{\sin(y)}\ dy$$
$$= \int [1 + \sin(y)\ ]dy = \int\ dy + \int \sin(y)\ dy = \boxed{y - \cos(y) + C}$$

(i)
$$\int \frac{1}{1+\sin(x)}\,dx$$

$$= \int \frac{1}{1+\sin(x)} \cdot \frac{1-\sin(x)}{1-\sin(x)}\,dx$$

$$= \int \frac{1-\sin(x)}{1-\sin^2(x)}\,dx$$

$$= \int \frac{1-\sin(x)}{\cos^2(x)}\,dx$$

$$= \int \frac{1}{\cos^2(x)} - \frac{\sin(x)}{\cos(x)\cos(x)}\,dx$$

$$= \int \sec^2(x)\,dx - \int \tan(x)\sec(x)\,dx$$

$$= \boxed{\tan(x) - \sec(x) + C}$$

6. A particle moves along an s-axis with position function $s = s(t)$ and velocity function $v(t) = s'(t)$. Use the given information to find $s(t)$.

(a) $v(t) = 32t; \quad s(0) = 20$

$$\begin{aligned} s(t) &= \int s'(t)\,dt = \int v(t)\,dt \\ &= \int 32t\,dt = 32\frac{t^2}{2} + C = 16t^2 + C \end{aligned}$$

$$\begin{aligned} s(0) &= 16(0)^2 + C = 20 \\ C &= 20 \end{aligned}$$

$$\boxed{s(t) = 16t^2 + 20}$$

(b) $v(t) = 3\sqrt{t}; \quad s(4) = 1$

$$\begin{aligned} s(t) &= \int v(t)\ dt \\ &= \int 3t^{1/2}\ dt = 3\frac{2}{3}t^{3/2} + C = 2t^{3/2} + C \end{aligned}$$

$$\begin{aligned} s(4) &= 2(4)^{3/2} + C = 1 \\ 16 + C &= 1 \\ C &= -15 \end{aligned}$$

$$\boxed{s(t) = 2t^{3/2} - 15}$$

7. Find an equation of the curve that satisfies the given conditions.

(a) At each point $(x,\ y)$ on the curve the slope is $2x + 1$; the curve passes through the point $(-3,\ 0)$.

What we are given is that the derivative of the curve is $2x + 1$, so we can find the curve by integrating:

$$\int 2x + 1\ dx = 2\frac{x^2}{2} + x + C = x^2 + x + C = f(x)$$

$$\begin{aligned} f(-3) &= (-3)^2 - 3 + C = 0 \\ 6 + C &= 0 \\ C &= -6 \end{aligned}$$

$$\boxed{f(x) = x^2 + x - 6}$$

(b) At each point $(x,\ y)$ on the curve the slope is $-\sin(x)$; the curve passes through the point $(0, 2)$

$$f(x) = \int -\sin(x)\ dx = \cos(x) + C$$

$$\begin{aligned} f(0) &= \cos(0) + C = 2 \\ 1 + C &= 2 \\ C &= 1 \end{aligned}$$

$$\boxed{f(x) = \cos(x) + 1}$$

(c) At each point $(x,\ y)$ on the curve, y satisfies the condition $d^2y/dx^2 = 6x$; the line $y = 5 - 3x$ is tangent to the curve at the point where $x = 1$.

First find $\frac{dy}{dx} = \int \frac{d^2y}{dx^2}\ dx = \int 6x\ dx = 3x^2 + C$.

At the point $x = 1$, the slope of the tangent line there is -3, hence $3(1)^2 + C = -3 \ \rightarrow\ C = -6$

So $\frac{dy}{dx} = 3x^2 - 6$. To find our curve $f(x)$ we just integrate $\frac{dy}{dx}$:

$$\int 3x^2 - 6\ dx = x^3 - 6x + C = f(x)$$

Now use that fact that at $x = 1,\ y = 5 - 3(1) = 2$,
so $f(1) = (1)^3 - 6(1) + C = 2 \ \rightarrow\ C = 7$

$$\boxed{f(x) = x^3 - 6x + 7}$$