Integration By Substitution
Solutions To Selected Problems
Calculus 9^{th} Edition Anton, Bivens, Davis

Matthew Staley

October 30, 2011

1. Evaluate the integrals using the indicated substitutions.

(a)

$$\int 2x(x^2+1)^{23}\ dx \quad u = x^2+1$$

$$du = 2x\ dx$$

$$\int u^{23}\ du = \frac{u^{24}}{24} + C = \boxed{\frac{(x^2+1)^{24}}{24} + C}$$

(b)

$$\int \cos^3(x)\sin(x)\ dx \quad u = \cos(x)\ dx$$

$$du = -\sin(x)\ dx$$

$$-du = \sin(x)\ dx$$

$$-\int u^3\ du = -\frac{u^4}{4} + C = \boxed{-\frac{\cos^4(x)}{4} + C}$$

(c)

$$\int \frac{1}{\sqrt{x}}\sin(\sqrt{x})\ dx \quad u = \sqrt{x}$$

$$du = \frac{1}{2\sqrt{x}}\ dx$$

$$2\ du = \frac{1}{\sqrt{x}}\ dx$$

$$2\int \sin(u)\ du = -2\cos(u) + C = \boxed{-2\cos(\sqrt{x}) + C}$$

(d)

$$\int \cot(x)\csc^2(x)\ dx \quad u = \cot(x)$$

$$du = -\csc^2(x)\ dx$$

$$-\ du = \csc^2(x)\ dx$$

$$-\int u\ du = -\frac{u^2}{2} + C = \boxed{-\frac{1}{2}\cot^2(x) + C}$$

(e) $$\int \cos(2x)\ dx \quad u = 2x$$

$$du = 2\ dx$$

$$\frac{1}{2}\ du = dx$$

$$\frac{1}{2}\int \cos(u)\ du = \frac{1}{2}\sin(u) + C = \boxed{\frac{1}{2}\sin(2x) + C}$$

(f) $$\int x^2\sqrt{1+x}\ dx \quad u = 1 + x \ \rightarrow\ u - 1 = x$$

$$du = dx$$

$$\begin{aligned}\int (u-1)^2\sqrt{u}\ du &= \int (u^2 - 2u + 1)u^{1/2}\ du \\ &= \int u^{5/2} - 2u^{3/2} + u^{1/2}\ du \\ &= \frac{2}{7}u^{7/2} - \frac{2}{5}2u^{5/2} + \frac{2}{3}u^{3/2} + C \\ &= \boxed{\frac{2}{7}(1+x)^{7/2} - \frac{4}{5}(1+x)^{5/2} + \frac{2}{3}(1+x)^{3/2} + C}\end{aligned}$$

2. Evaluate the integrals using appropriate substitutions.

(a) $$\int (4x-3)^9\ dx \quad Let\ u = 4x - 3$$

$$du = 4\ dx$$

$$\frac{1}{4}du = dx$$

$$\frac{1}{4}\int u^9\ du = \frac{1}{4}\frac{u^10}{10} + C = \boxed{\frac{1}{40}(4x-3)^{10} + C}$$

(b)

$$\int \sec(4x)\tan(4x)\ dx \quad \textit{Let } u = 4x$$

$$du = 4\ dx$$

$$\frac{1}{4}du = dx$$

$$\frac{1}{4}\int \sec(u)\tan(u)\ du = \frac{1}{4}\sec(u) + C = \boxed{\frac{1}{4}\sec(4x) + C}$$

(c)

$$\int \frac{\sin\left(\frac{5}{x}\right)}{x^2}\ dx \quad \textit{Let } u = \frac{5}{x}$$

$$du = -\frac{5}{x^2}\ dx$$

$$-\frac{1}{5}du = \frac{1}{x^2}\ dx$$

$$-\frac{1}{5}\int \sin(u)\ du = -\frac{1}{5}(-\cos(u)) + C = \boxed{\frac{1}{5}\cos(5/x) + C}$$

(d)

$$\int x\sec^2(x^2)\ dx \quad \textit{Let } u = x^2$$

$$du = 2x\ dx$$

$$\frac{1}{2}dx = x\ du$$

$$\frac{1}{2}\int \sec^2(u)\ du = \frac{1}{2}\tan(u) + C = \boxed{\frac{1}{2}\tan(x^2) + C}$$

(e)

$$\int \cos(4x)\sqrt{2-\sin(4x)}\ dx \quad \textit{Let } u = 2-\sin(4x)$$

$$du = -4\cos(4x)\ dx$$

$$-\frac{1}{4} = \cos(4x)\ dx$$

$$-\frac{1}{4}\int \sqrt{u}\ du = -\frac{1}{4}\int u^{1/2}\ du$$

$$= -\frac{1}{4}\frac{2}{3}u^{3/2} + C = \boxed{-\frac{1}{6}(2-\sin(4x))^{3/2} + C}$$

(f)

$$\int \frac{y}{\sqrt{2y+1}}\ dy \quad \textit{Let } u = 2y+1 \ \rightarrow \ \frac{1}{2}(u-1) = y$$

$$du = 2\ dy$$

$$\frac{1}{2}du = dy$$

$$\frac{1}{2}\ \int \frac{\frac{1}{2}(u-1)}{\sqrt{u}}\ du = \frac{1}{4}\int \frac{u}{\sqrt{u}} - \frac{1}{\sqrt{u}}\ du$$

$$= \frac{1}{4}\int u^{1/2} - u^{-1/2}\ du$$

$$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right] + C$$

$$= \boxed{\frac{1}{6}(2y+1)^{3/2} - \frac{1}{2}(2y+1)^{1/2} + C}$$