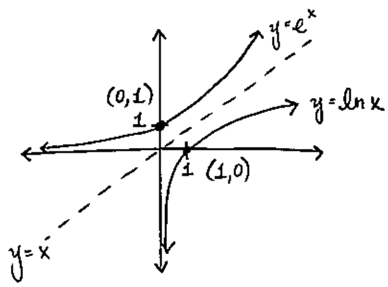
SECTION 0.1 FUNCTIONS



$x = \ln_e y$

Recall:

$\log_8 2 = \frac{1}{3}$

$8^{\frac{1}{3}} = 2$

$\sqrt[3]{8} = 2$

$2 = 2$ ✓

$\left.\begin{array}{l} D: \mathbb{R} \\ R: (0, +\infty) \end{array}\right\} y = e^x$ $\quad$ $\left.\begin{array}{l} D: (0, +\infty) \\ R: \mathbb{R} \end{array}\right\} y = \ln x$

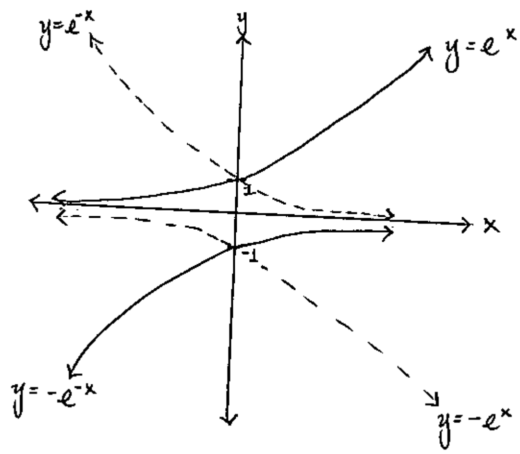
Note: Take notice that $y = e^x$ and $y = \ln x$ are inverse of one another as are their domains (D) & ranges (R).

* $\mathbb{R}$ - means all Real #'s

? Can $\ln x$ equal zero?

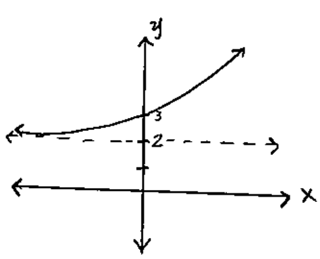
A: Yes, if $x = 1$

$\ln 1 = 0$



Graph & find D & R.

$y = e^x + 2$



D: $\mathbb{R}$

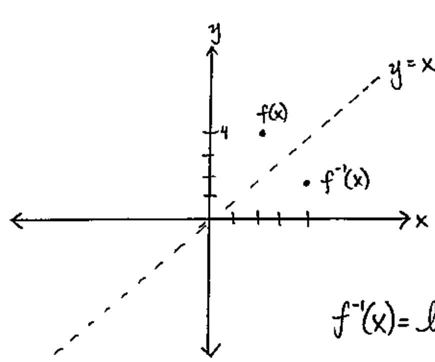
R: $(2, +\infty)$

What is the D & R of $f^{-1}(x)$ given $f(x) = e^{x-2} - 4$.

$f(x) = e^{x-2} - 4$

2 units to the Right

downward 4units



$f^{-1}(x) = \ln(x+4) + 2$

f(x) D: $\mathbb{R}$

R: $(-4, +\infty)$

$f^{-1}(x)$ D: $(-4, \infty)$

R: $\mathbb{R}$

E1 Graph $y = |2x - 1| + 1$

Solve for x & y for each piecewise.

$2x - \cancel{1} \geq 0$ ▶ $y = 2x - 1 + 1$

$\cancel{+1}$ $+1$ $\boxed{y = 2x}$

$\frac{\cancel{2}x}{\cancel{2}} \geq \frac{1}{2}$

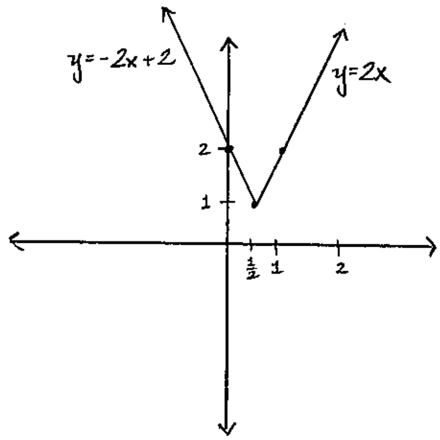
$\boxed{x \geq \frac{1}{2}}$

$2x - \cancel{1} < 0$ ▶ $y = -[2x - 1] + 1$

$\cancel{+1}$ $+1$ $y = -2x + 1 + 1$

$\frac{\cancel{2}x}{\cancel{2}} < \frac{1}{2}$ $\boxed{y = -2x + 2}$

$\boxed{x < \frac{1}{2}}$

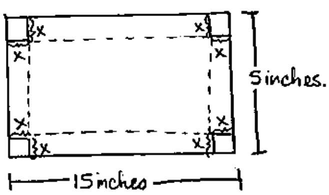


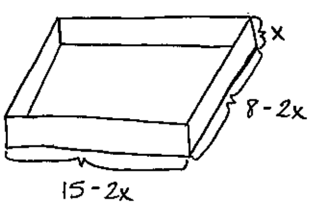
Note:

Absolute Values:

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x \leq 0 \end{cases}$$

E2 Section 0.1 #29





$V = lwh$

$V = (15 - 2x)(8 - 2x)(x)$

$V = (120 - 30x - 16x + 4x^2)x$

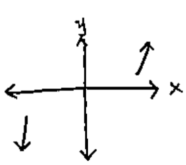
$V = 4x^3 - 46x^2 + 120x$

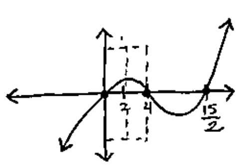
$V = 2x(2x^2 - 23x + 60)$

$V = 2x(2x^2 - 8x)(-15x + 60)$

$V = 2x[2x(x-4)] - 15(x-4)$

$V = 2x(2x - 15)(x - 4)$





$x = 0, \frac{15}{2}, 4$

D: (0,4)

2 x-intercepts max. (Plug 2 for x)

$* \ 15 - 2x = 0$

$x = \frac{15}{2}$

$* \ 8 - 2x = 0$

$x = 4$

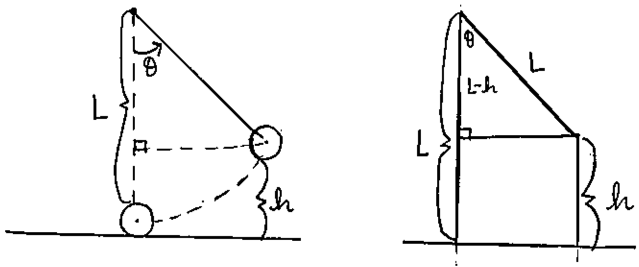
$V = lwh \quad (x = 2)$

$V = 2[15 - 2(2)][8 - 2(2)]$

$V = 2 \cdot 11 \cdot 4 \text{ in}^3$

$V = 88 \text{ in}^3$

E3 #25



$h(\theta) = ?$

$$\cos\theta = \frac{L-h}{L}$$

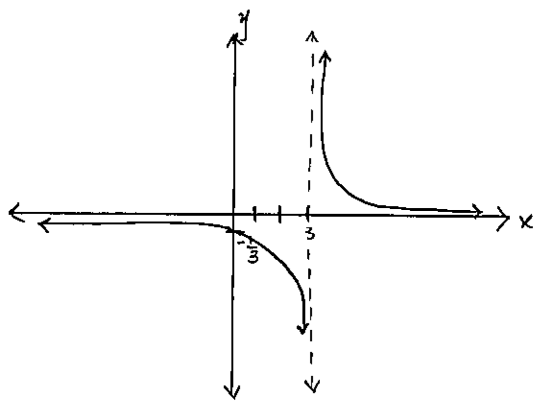
$$L\cos\theta = L - h$$

$$h = L - L\cos\theta$$

$$\boxed{h = L(1-\cos\theta)}$$

E4 9A.

$$f(x) = \frac{1}{x-3}$$



$f(x) = 0$

$f(0) = \frac{1}{x-3}$

$f(0) = \frac{1}{0-3}$

$f(0) = -\frac{1}{3}$

$x - 3 = 0$

$x = 3$

V.A. $x = 3$

* V.A. - vertical asymptote.

$$\boxed{\begin{array}{l} D: \mathbb{R} \quad x \neq 3 \\ R: \mathbb{R} \quad y \neq 0 \end{array}}$$

E5 Sketch:

$$f(x) = \frac{(x-2)(x+4)^2}{(x-3)^3(x+1)^2}$$

$$f(0) = \frac{(-2)(16)}{(-27)(1)}$$

$$f(0) = \frac{32}{27} = 1.18 = y\text{-intercept}$$

Note:

Degree : 3

Degree : 5

$3 < 5$

V.A. $x = 3$

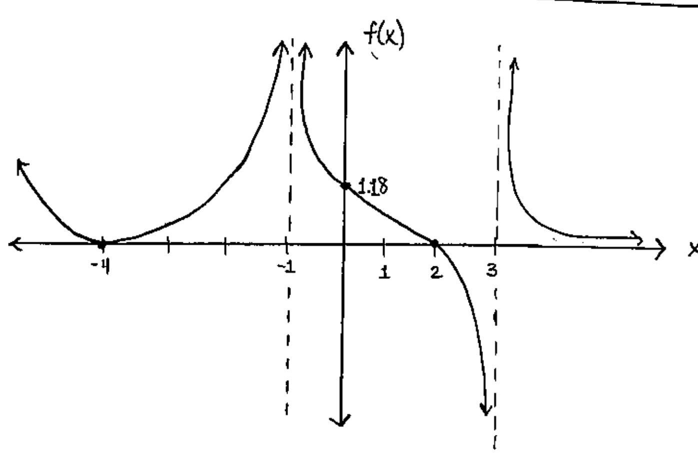
$x = -1$

* if degrees vary by more than 1 than no H.A. (horizontal asymptote) exists.

* if the denominator power is larger than the numerator by 1 then H.A. $y = 0$, if the numerator is larger than the denominator by 1, then a slant asymptote (S.A.) exists.

Sign Chart

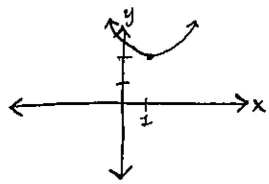
	$x<-4$	$-4<x<-1$	$-1<x<0$	$0<x<2$	$2<x<3$	$x>3$
$(x-2)$	(−)	(−)	(−)	(−)	(+)	(+)
$(x+4)^2$	(+)	(+)	(+)	(+)	(+)	(+)
$(x-3)^3$	(−)	(−)	(−)	(−)	(−)	(+)
$(x+1)^2$	(+)	(+)	(+)	(+)	(+)	(+)
	(+)	(+)	(+)	(+)	(−)	(+)
	Above	Above	Above	Above	Below	Above



$\boxed{E6}$ Find f composition of g [$f \circ g$ -OR- $f[g(x)]$]

$$f[g(x)] = (x-1)^2 + 2$$
$$f[g(x)] = x^2 - 2x + 1 + 2$$
$$f[g(x)] = x^2 - 2x + 3$$

Vertex $(1, 2)$



SECTION 0.2 COMPOSITION OF FUNCTIONS

$\boxed{E1}$

$f(x) = x^2 + 2$	$g(x) = x - 1$
SHAPE: parabola	SHAPE: line
y-int. @ $y = 2$	y-int. @ $y = -1$
$\mathbb{D}: (2, \infty)$	$\mathbb{D}: \mathbb{R}$

Recall

STANDARD FORMULA

$$ax^2 + bx + c = 0$$

QUADRATIC FORMULA

$$\frac{(-b) \pm \sqrt{(b^2) - 4ac}}{2a} = x$$

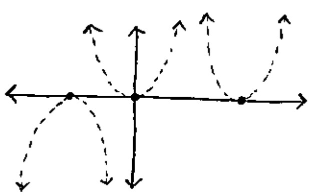
DISCRIMINANT

$$\sqrt{(b^2) - 4ac}$$

if the discriminant of a standard formula is:

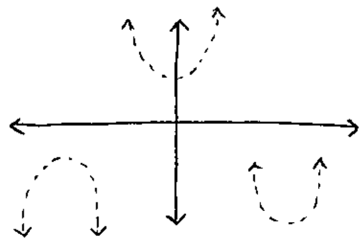
$\boxed{\sqrt{(b^2) - 4ac} = 0}$

- the parabola will touch the x-axis once.



$\boxed{\sqrt{(b^2) - 4ac} < 0}$

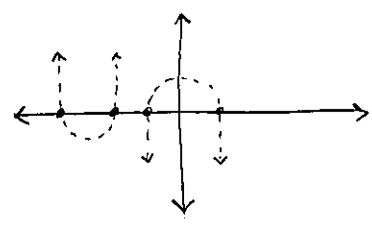
- the parabola will NOT touch the x-axis.



* Imaginary #'s.

$\boxed{\sqrt{(b^2) - 4ac} > 0}$

- the parabola will touch the x-axis twice.



$\boxed{E2}$ Express $h(x) = (x+2)^2$ as a composition of two functions.

$f(x) = x^2$

$g(x) = (x+2)$

$$\boxed{f \circ g = h(x)}$$

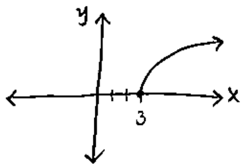
SECTION 0.2 cont.

Recall Graphing & Determining Domain & Range of Functions

- $f(x) = \sqrt{x-3}$

$D: [3, +\infty)$

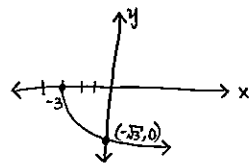
$R: [0, +\infty)$



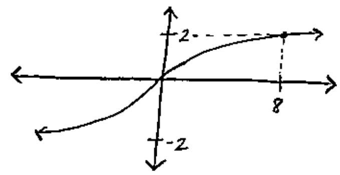
- $g(x) = -\sqrt{x-3}$

$D: [-3, +\infty)$

$R: (-\infty, 0]$



- $h(x) = \sqrt[3]{x}$



$\boxed{E3}$ SECTION 0.2 #17.

$$y = |x+2| - 2$$

$$y = \begin{cases} x \;;\; x \geq -2 \\ -x-4 \;;\; x < -2 \end{cases}$$

$|x+2| - 2$

If $x+2 \geq 0$, then x must be positive.

$x+2-2 = 0$

$x = 0$

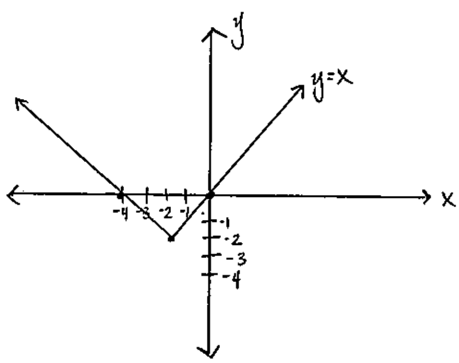
If $x+2 < 0$, then x must be negative.

$-(x+2) - 2 = 0$

$-x-2-2 = 0$

$-x-4 = 0$

$x = -4$



$$|x| = \begin{cases} x \;;\; x \geq 0 \\ -x \;;\; x < 0 \end{cases}$$

$\boxed{E4}$ SECTION 0.2 #31.

$f(x) = x^2$

$g(x) = \sqrt{1-x}$

$f \circ g = f[g(x)]$

$= (\sqrt{1-x})^2$

$= 1 - x$

$= -x + 1$

D: $\mathbb{R}$

R: $\mathbb{R}$

$g \circ f = g[f(x)]$

$= \sqrt{1-(x^2)}$

$= \sqrt{1-x^2}$

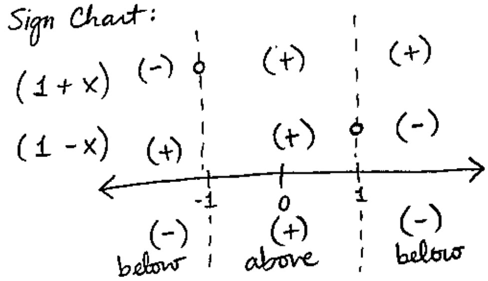
$D: [-1, 1]$

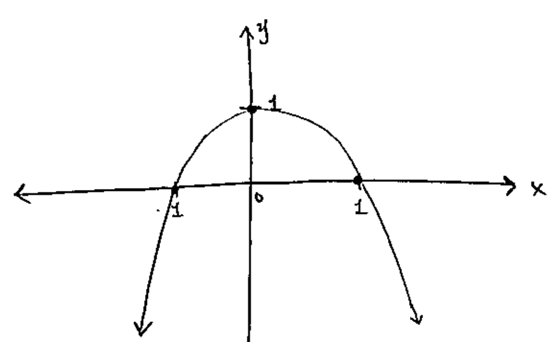
$R: [0, 1]$

$y = 1 - x^2$

$y = (1+x)(1-x)$

Sign Chart:





SECTION 0.2 cont.

$\boxed{E5}$ SECTION 0.2 #29.

$f(x) = \sqrt{x}$ $\qquad$ $g(x) = x^3 + 1$

a.) Find $f[g(2)]$:

$$f[g(2)] = \sqrt{x^3+1}$$
$$= \sqrt{(2)^3+1}$$
$$= \sqrt{8+1}$$
$$= \pm\sqrt{9}$$
$$\boxed{f[g(2)] = 3}$$

e.) $f(2+h)$

$$\boxed{f(2+h) = \sqrt{2+h}}$$

f.) $g(3+h)$

$$\boxed{g(3+h) = (3+h)^3 + 1}$$

Recall: PASCAL'S TRIANGLE

Row 0 $\qquad$ 1

Row 1 $\qquad$ 1 $\quad$ 1

Row 2 $\qquad$ 1 $\quad$ 2 $\quad$ 1

Row 3 $\qquad$ 1 $\quad$ 3 $\quad$ 3 $\quad$ 1

Row 4 $\qquad$ 1 $\quad$ 4 $\quad$ 6 $\quad$ 4 $\quad$ 1

Row 5 $\qquad$ 1 $\quad$ 5 $\quad$ 10 $\quad$ 10 $\quad$ 5 $\quad$ 1

Row 6 $\qquad$ 1 $\quad$ 6 $\quad$ 15 $\quad$ 20 $\quad$ 15 $\quad$ 6 $\quad$ 1

Row 7 $\qquad$ 1 $\quad$ 7 $\quad$ 21 $\quad$ 35 $\quad$ 35 $\quad$ 21 $\quad$ 7 $\quad$ 1

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$\vdots$

TRY

$(a+b)^7$:

$\boxed{}$

Pascal's triangle can be expanded infinitely when a binomial is raised to a "n^{th}" power. Each row represents the coefficients of the n^{th} expanded binomial.

Ex. $(x+y)^3$

$$1(x^3) + 3(x^2y) + 3(xy^2) + 1(y^3)$$
$$\boxed{x^3 + 3x^2y + 3xy^2 + y^3}$$

Ex. $(3+h)^3$

$$1(3^3 \cdot h^0) + 3(3^2 \cdot h) + 3(3^1 \cdot h^2) + 1(3^0 \cdot h^3)$$
$$\boxed{27 + 27h + 9h^2 + h^3}$$

$\boxed{E6}$ SECTION 0.2 #39.A. $\qquad$ $f(x) = \sin^2 x$

$g(x) = x^2$ $\qquad$ $h(x) = \sin x$

$f(x) = g[h(x)]$

$$f(x) = g[h(x)]$$
$$f(x) = (\sin x)^2$$
$$f(x) = \sin^2 x$$

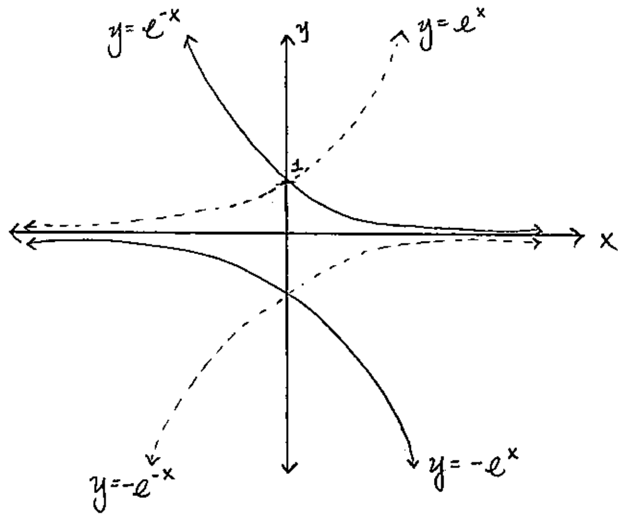
SECTION 0.3 FUNCTIONS

$\boxed{y = \frac{1}{e^x} = e^{-x}}$ ⟷

D: $\mathbb{R}$
R: $(0, +\infty)$

$\boxed{y = -\frac{1}{e^x} = -e^{-x}}$ ⟵ - - ⟶

D: $\mathbb{R}$
R: $(-\infty, 0)$



$\boxed{y = e^x}$ ⟵ - - ⟶

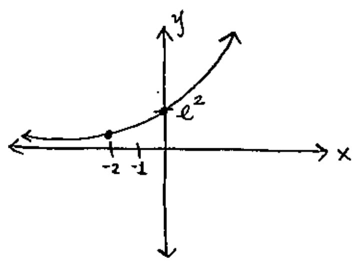
$1 = e^0$

D: $\mathbb{R}$
R: $(0, +\infty)$

$\boxed{y = -e^x}$ ⟷

D: $\mathbb{R}$
R: $(-\infty, 0)$

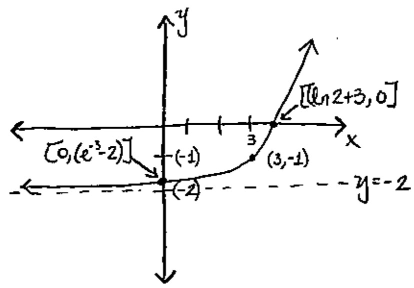
$\boxed{y = e^{x+2}}$



x	y
0	e^2

y-int: $(0, e^2)$

$\boxed{y = e^{x-3} - 2}$



x-intercept (set y=0)

$0 = e^{x-3} - 2$

$2 = e^{x-3}$

$\ln 2 = (x-3)\cancel{\ln e}^{1}$

$\ln 2 = x - 3$

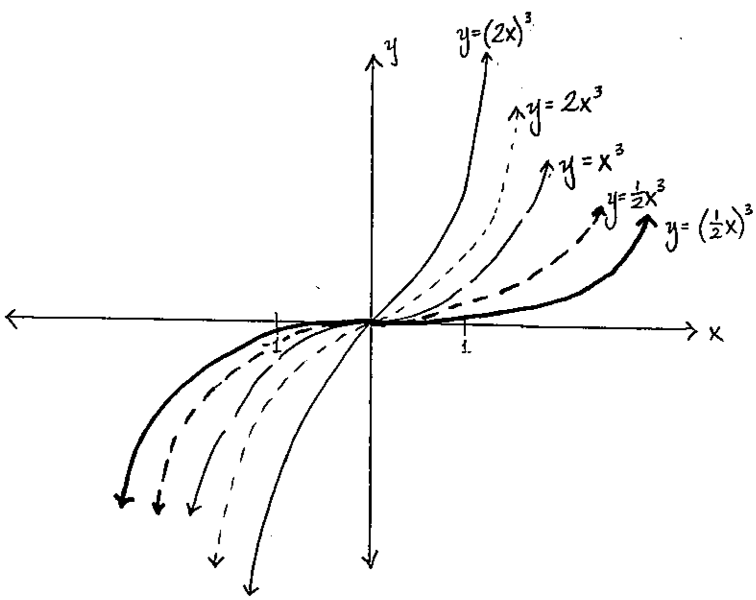
$x = \ln 2 + 3$

$[(\ln 2 + 3), 0]$

y-intercept (set x=0)

x	y
0	$e^{-3} - 2$

$[0, (e^{-3} - 2)]$



TRANSFORMATIONS

When a function [Ex. $f(x) = x^3$] is multiplie
by a constant. The graph of that functi
will stretch or compress by that constan

Horizontal Compression
$y = (2x)^3 > 1$

Horizontal Stretch
$y = (\frac{1}{2}x)^3 < 1$

Vertical Compression
$y = \frac{1}{2}x^3 < 1$

Vertical Stretch
$y = (2x)^3 > 1$

SECTION 0.3 cont.

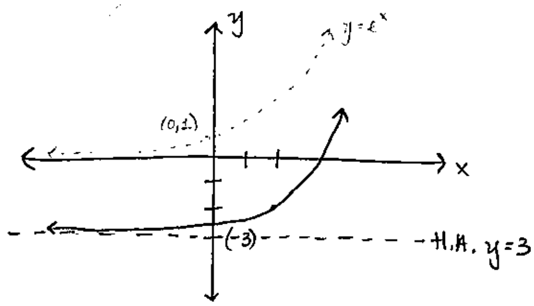
$\boxed{E1}$ Give the Domain & Range (D&R) of

$$f(x) = e^{x-2} - 3$$

NOTE: $f(x) = e^{x-2} - 3$

shifts graph to the right 2 units

Horizontal Asymptote (H.A.) @ $y = -3$



D: $\mathbb{R}$
R: $(-3, +\infty)$

$\boxed{E2}$ Find the D&R of

$$y = |2x-1| + 1$$

Shifts: Right $\frac{1}{2}$
Up 1

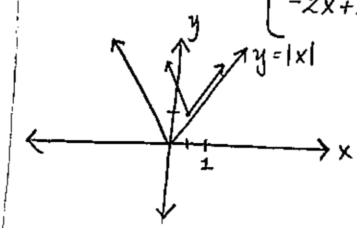
$2x - 1 + 1 = y$
$y = 2x$

$-2x + 1 + 1 = y$
$y = -2x + 2$

$2x - 1 \geq 0$
$2x \geq 1$
$x \geq \frac{1}{2}$

$2x - 1 < 0$
$2x < 1$
$x < \frac{1}{2}$

$$y = |2x-1| + 1 \begin{cases} 2x \;; & x \geq \frac{1}{2} \\ -2x+2 \;; & x < \frac{1}{2} \end{cases}$$



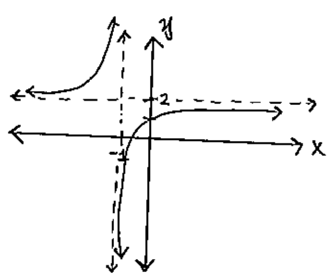
D: $\mathbb{R}$
R: $[1, +\infty)$

*Example of Horizontal Compression.

$\boxed{E3}$ $y = 2 - \frac{1}{x+1}$

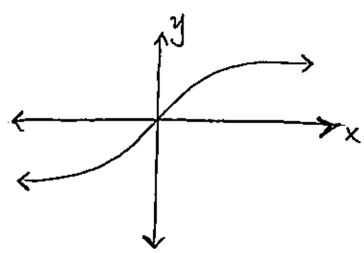
up 2 units

Left 1 unit



D: $\mathbb{R}$; $x \neq -1$ – OR – $(-\infty, -1) \cup (-1, +\infty)$
R: $\mathbb{R}$; $y \neq 2$ – or – $(-\infty, 2) \cup (2, +\infty)$

$\boxed{E4}$ $y = \sqrt[5]{x}$

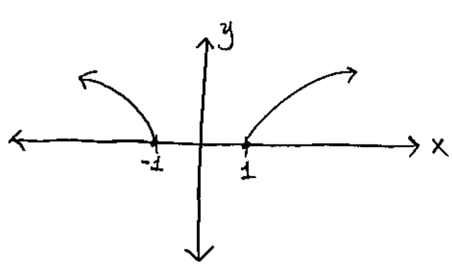


D: $\mathbb{R}$
R: $\mathbb{R}$

$\boxed{E5}$ $y = \sqrt{x^2 - 1}$

$x^2 - 1 = (x-1)(x+1)$

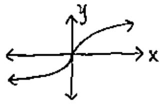
D: $(-\infty, -1] \cup [1, +\infty)$
R: $[0, +\infty)$



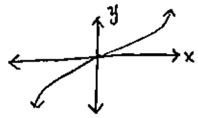
SECTION 0.3 cont.

$\boxed{E6}$ SECTION 0.3 #11.

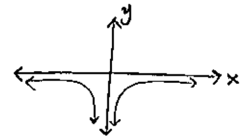
A. $y = \sqrt[5]{x}$



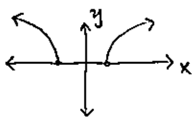
B. $y = 2x^5$



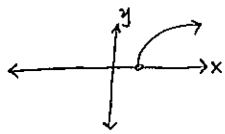
C. $y = \frac{1}{x^8}$



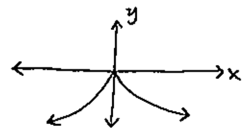
D. $y = \sqrt{x^2-1}$



E. $y = \sqrt[4]{x-2}$

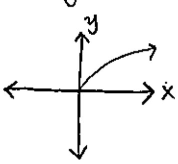


F. $y = -\sqrt[5]{x^2}$



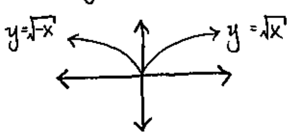
$\boxed{E7}$ SECTION 0.3 #20.A

$y = \sqrt{x}$



* "x" cannot be negative.

$y = \sqrt{|x|}$



* "x" can be negative or positive.

$f(x) = f^{-1}(x)$ inverse

$\boxed{E8}$ SECTION 0.3 # 29A.

$$y = \frac{x^2}{x^2-x-2}$$

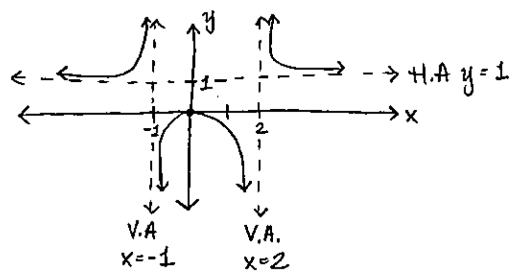
$$y = \frac{x^2}{(x-2)(x+1)}$$

H.A. $\frac{x^2}{x^2} = \frac{1}{1} = 1$

$y = 1$

V.A.

$x = -1, 2$



D: $\mathbb{R}$ $x \neq -1, 2$

R: $(-\infty, 0] \cup (1, +\infty)$

A. II	C. IV
B. I	D. III

Recall:

CAUTION!! NOT EQUIVALENT!

-2^2	$(-2)^2$
$(-1)\cdot 2^2$	$(-2)\cdot(-2)$
-4	4

$\sqrt{-x}$	$\sqrt{(-x)}$
$\sqrt{(-1)\cdot x}$	$\sqrt{-x}$