Calculus I

Sample Exam #03

1. Integrate the following. Rationalize the denominator if necessary and write your solution as a single fraction.

   a) $\int_{\pi/6}^{\pi/3} \sin x \cos x \; dx$

   b) $\int_{\pi/6}^{2\pi/3} \frac{\cos x}{\sin^2 x} \; dx$

2. Integrate the following. Rationalize the denominator if necessary and write your solution as a single fraction.

a) $\int_{\pi/4}^{\pi/3} \tan x \sec^2 x \; dx$

b) $\int_{\pi/6}^{2\pi/3} \frac{\sin x}{\cos^2 x} \; dx$

3. Evaluate the integral by making the appropriate substitutions. Write your solution as a single fraction.

a) $\displaystyle\int_{\frac{4\pi^2}{9}}^{\frac{16\pi^2}{9}} \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx$

b) $\displaystyle\int_0^3 \frac{x^2}{\sqrt{x+1}}\,dx$

4. Given, $\int_{-2}^{3} f(x)\,dx = -2$, $\int_{-2}^{3} g(x)\,dx = -4$, $\frac{2}{3}\int_{-2}^{3} h(x)\,dx = 1$, find the following:

a) $2\int_{-2}^{3}[g(x)-2x]\,dx$

b) $\int_{-2}^{3}\left[2f(x)-\frac{3}{2}h(x)\right]dx$

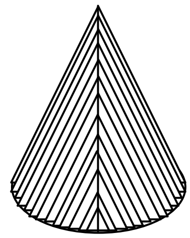
5. A particle moves along the s-axis. Use the given information to find the position function of the particle.

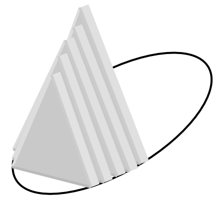
$$a(t) = 1+\cos 2t\ ; \quad v(\pi) = 0; \quad s\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}$$

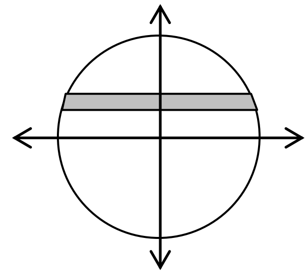
6. A particle moves with acceleration $a(t)\ m/s^2$ along an s-axis and has velocity $v_0\ m/s$ at time $t=0$. Find the displacement and the distance traveled by the particle during the stated time interval.

$a(t)=6t-4$; $v(0)=1$; $0\le t\le 1$

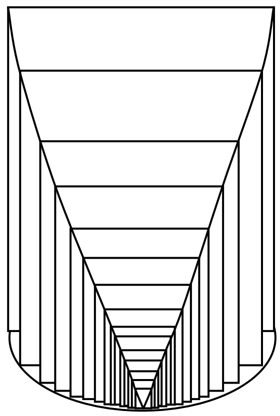
7. A solid has a circular base with equation $x^2 + y^2 = 4$. Parallel cross-sections perpendicular to the y-axis are equilateral triangles. Find the volume.



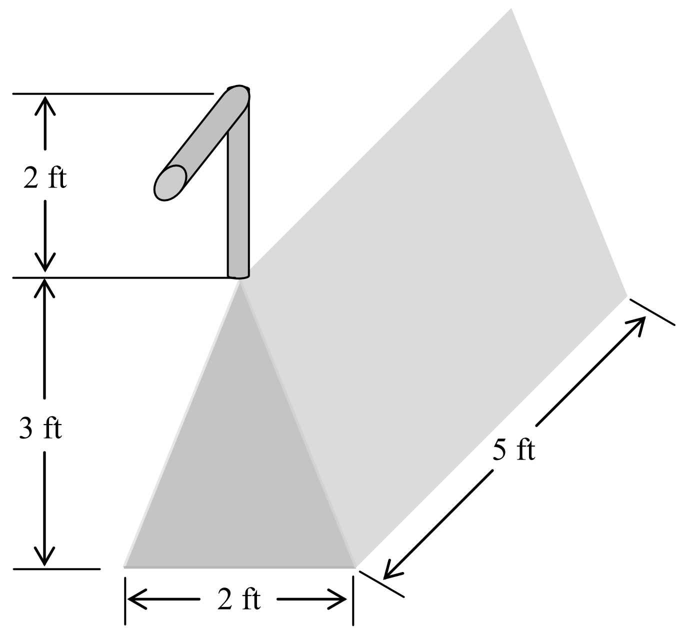




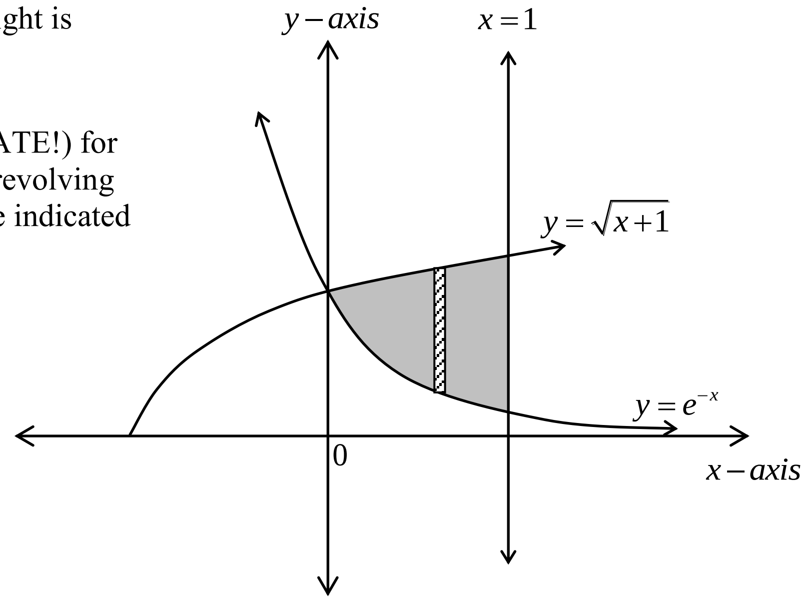
8. A solid has a circular base with equation $x^2 + y^2 = 4$. Parallel cross-sections perpendicular to the x-axis are rectangles **whose height is twice the length of its width**. Find the volume.



9. The tank below is full of heating oil with density 100 lb/ft^3. Find the work required to pump the oil out the outlet. See figure.



10. The region shown in the figure to the right is bounded by the graphs;
$y = e^{-x}$, $y = \sqrt{x+1}$, $x = 0$ and $x = 1$
Write the integral (DO NOT INTEGRATE!) for the volume V of the solid obtained by revolving the region about the stated line. Use the indicated method.



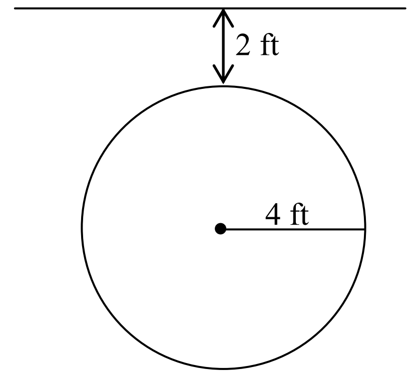
a) $y = -1$; Washer Method

b) $x = -1$; Shell Method

c) $y = 3$; Washer Method

d) $x = 2$; Shell Method

11. The flat surface shown is submerged in a fluid. Find the fluid force against the surface. The fluid density is $10\dfrac{lbs}{ft^3}$.



Calculus I

Sample Exam #03

1. Integrate the following. Rationalize the denominator if necessary and write your solution as a single fraction.

a) $\int_{\pi/6}^{\pi/3} \sin x \cos x \; dx$

$u = \sin x$

$du = \cos x \, dx$

$x = \frac{\pi}{3} \rightarrow u = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6} \rightarrow u = \sin\frac{\pi}{6} = \frac{1}{2}$

$$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} u \, du$$

$$= \frac{u^2}{2}\Bigg|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2}\left[u^2\Big|_{1/2}^{\sqrt{3}/2}\right]$$

$$= \frac{1}{2}\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right]$$

$$= \frac{1}{2}\left[\frac{3}{4} - \frac{1}{4}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{4}\right]$$

$$= \boxed{\frac{1}{4}}$$

b) $\int_{\pi/6}^{2\pi/3} \frac{\cos x}{\sin^2 x} \; dx$

$u = \sin x$

$du = \cos x \, dx$

$x = \frac{2\pi}{3} \rightarrow u = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6} \rightarrow u = \sin\frac{\pi}{6} = \frac{1}{2}$

$$\int_{1/2}^{\sqrt{3}/2} \frac{1}{u^2} \, du$$

$$= \int_{1/2}^{\sqrt{3}/2} u^{-2} \, du$$

$$= -\frac{1}{u}\Bigg|_{1/2}^{\sqrt{3}/2}$$

$$= \frac{-1}{\frac{\sqrt{3}}{2}} - \left(\frac{-1}{\frac{1}{2}}\right)$$

$$= -\frac{2}{\sqrt{3}} + 2$$

$$= \frac{-2\sqrt{3}}{3} + \frac{6}{3}$$

$$= \boxed{\frac{6 - 2\sqrt{3}}{3}}$$

2. Integrate the following. Rationalize the denominator if necessary and write your solution as a single fraction.

a) $\int_{\pi/4}^{\pi/3} \tan x \sec^2 x \; dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$x = \frac{\pi}{3} \rightarrow u = \tan\frac{\pi}{3} = \sqrt{3}$

$x = \frac{\pi}{4} \rightarrow u = \tan\frac{\pi}{4} = 1$

$$\int_1^{\sqrt{3}} u \, du$$

$$= \frac{u^2}{2}\Big|_1^{\sqrt{3}}$$

$$= \frac{(\sqrt{3})^2}{2} - \frac{(1)^2}{2}$$

$$= \frac{3}{2} - \frac{1}{2}$$

$$= \frac{2}{2}$$

$$= \boxed{1}$$

b) $\int_{\pi/6}^{2\pi/3} \frac{\sin x}{\cos^2 x} \; dx$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$x = \frac{2\pi}{3} \rightarrow u = \cos\frac{2\pi}{3} = -\frac{1}{2}$

$x = \frac{\pi}{6} \rightarrow u = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$$-\int_{\sqrt{3}/2}^{-1/2} u^{-2} \, du$$

$$= \int_{-1/2}^{\sqrt{3}/2} u^{-2} \, du$$

$$= -\frac{1}{u}\Big|_{-1/2}^{\sqrt{3}/2}$$

$$= \left(-\frac{2}{\sqrt{3}}\right) - (2)$$

$$= \frac{-2\sqrt{3}}{3} - \frac{6}{3}$$

$$= \boxed{\frac{-2\sqrt{3} - 6}{3}}$$

3. Evaluate the integral by making the appropriate substitutions. Write your solution as a single fraction.

a) $\int_{\frac{4\pi^2}{9}}^{\frac{16\pi^2}{9}} \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$x = \frac{16\pi^2}{9} \rightarrow u = \sqrt{\frac{16\pi^2}{9}} = \frac{4\pi}{3}$

$x = \frac{4\pi^2}{9} \rightarrow u = \sqrt{\frac{4\pi^2}{9}} = \frac{2\pi}{3}$

$2\int_{2\pi/3}^{4\pi/3} \sec^2 u\,du$

$= 2\tan u \Big|_{2\pi/3}^{4\pi/3}$

$= 2\left[\tan\frac{4\pi}{3} - \tan\frac{2\pi}{3}\right]$

$= 2\left[\sqrt{3} - (-\sqrt{3})\right]$

$= 2\left[2\sqrt{3}\right]$

$= \boxed{4\sqrt{3}}$

b) $\int_0^3 \frac{x^2}{\sqrt{x+1}}\,dx$

$u = x+1$

$x = u-1$

$x^2 = u^2 - 2u + 1$

$du = dx$

$x = 3 \rightarrow u = 4$

$x = 0 \rightarrow u = 1$

$\int_1^4 \frac{u^2-2u+1}{\sqrt{u}}\,du$

$= \int_1^4 \left(u^{3/2} - 2u^{1/2} + u^{-1/2}\right)du$

$= \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + 2u^{1/2}\Big|_1^4$

$= \left(\frac{2}{5}\cdot 32 - \frac{4}{3}\cdot 8 + 2\cdot 2\right) - \left(\frac{2}{5} - \frac{4}{3} + 2\right)$

$= \frac{64}{5} - \frac{32}{3} + 4 - \frac{2}{5} + \frac{4}{3} - 2$

$= \frac{62}{5} - \frac{28}{3} + 2$

$= \frac{186}{15} - \frac{140}{15} + \frac{30}{15}$

$= \frac{46}{15} + \frac{30}{15}$

$= \boxed{\frac{76}{15}}$

4. Given, $\int_{-2}^{3} f(x)\,dx = -2$, $\int_{-2}^{3} g(x)\,dx = -4$, $\frac{2}{3}\int_{-2}^{3} h(x)\,dx = 1$, find the following:

$\int_{-2}^{3} h(x)dx = \frac{3}{2}$

a) $2\int_{-2}^{3}[g(x)-2x]\,dx$

$2\int_{-2}^{3} g(x)dx - 2\int_{-2}^{3} 2x\,dx$

$= 2(-4) - 2\left[x^2\Big|_{-2}^{3}\right]$

$= -8 - 2[9-4]$

$= -8 - 2(5)$

$= -8 - 10$

$= \boxed{-18}$

b) $\int_{-2}^{3}\left[2f(x)-\frac{3}{2}h(x)\right]dx$ $= \frac{3}{2}\int_{-2}^{3} h(x)dx = \frac{9}{4}$

$2\int_{-2}^{3} f(x)dx - \frac{3}{2}\int_{-2}^{3} h(x)dx$

$= 2(-2) - \frac{3}{2}\left(\frac{3}{2}\right)$

$= -4 - \frac{9}{4}$

$= -\frac{16}{4} - \frac{9}{4}$

$= \boxed{-\frac{25}{4}}$

5. A particle moves along the s-axis. Use the given information to find the position function of the particle.

$a(t) = 1+\cos 2t$; $v(\pi)=0$; $s\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}$

$v(t) = \int a(t)\,dt$

$v(t) = \int (1+\cos 2t)\,dt$

$v(\pi) = t + \frac{1}{2}\sin 2t + C$

$0 = \pi + \cancel{\frac{1}{2}\sin 2\pi}^{0} + C$

$-\pi = C$

$v(t) = t + \frac{1}{2}\sin 2t - \pi$

$s(t) = \int v(t)\,dx$

$s(t) = \int \left(t + \frac{1}{2}\sin 2t - \pi\right)dt$

$s\left(\frac{\pi}{4}\right) = \frac{t^2}{2} - \frac{1}{4}\cos 2t - \pi t + C$

$\frac{\pi^2}{32} = \frac{\left(\frac{\pi}{4}\right)^2}{2} - \frac{1}{4}\cos\frac{\pi}{2} - \pi\left(\frac{\pi}{4}\right) + C$

$\frac{\pi^2}{32} = \frac{\pi^2}{32} - 0 - \frac{\pi^2}{4} + C$

$\frac{\pi^2}{4} = C$

$\boxed{s(t) = \frac{t^2}{2} - \frac{1}{4}\cos 2t - \pi t + \frac{\pi^2}{4}}$

6. A particle moves with acceleration $a(t)\ m/s^2$ along an s-axis and has velocity $v_0\ m/s$ at time $t = 0$. Find the displacement and the distance traveled by the particle during the stated time interval.

$a(t) = 6t - 4$; $v(0) = 1$; $0 \le t \le 1$

$V(t) = \int a(t)\,dt$

$v(t) = \int 6t - 4$

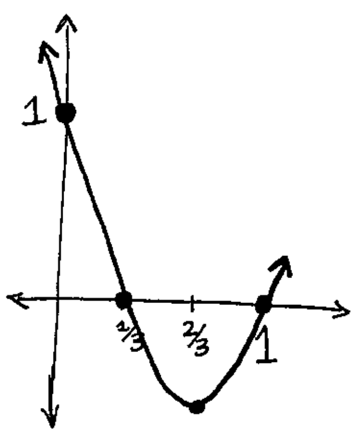
$V(0) = 3t^2 - 4t + C$

$1 = 3(0)^2 - 4(0) + C$

$1 = C$

$v(t) = 3t^2 - 4t + 1$

$V(t) = (3t - 1)(t - 1)$



$s(t) = \int_0^{1/3} (3t^2 - 4t + 1)\,dt$

$= \frac{\cancel{3}t^3}{\cancel{3}} - \frac{\cancel{4}^{2}t^2}{\cancel{2}} + t\Big|_0^{1/3}$

$= t^3 - 2t^2 + t\Big|_0^{1/3}$

$= \left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right) - (0)$

$= \frac{1}{27} - \frac{2}{9} + \frac{1}{3} - 0$

$= \frac{1}{27} - \frac{6}{27} - \frac{9}{27}$

$= \frac{4}{27}$

$s(t) = \int_{1/3}^{1} (3t^2 - 4t + 1)\,dt$

$= t^3 - 2t^2 + t\Big|_{1/3}^{1}$

$= (1 - 2 + 1) - \left(\frac{4}{27}\right)$

$= -\frac{4}{27}$

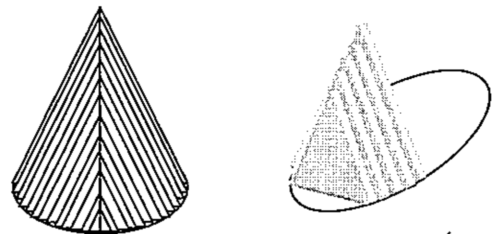
DISTANCE:

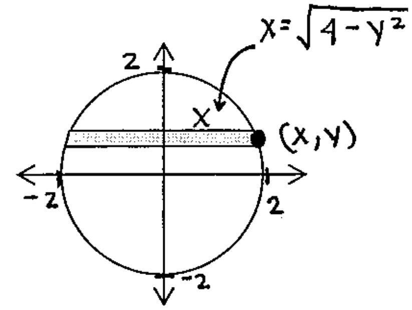
$\int_0^1 |v(t)|\,dt = \left|\frac{4}{27}\right| + \left|\frac{-4}{27}\right| =$ $\boxed{\text{DISTANCE: } \frac{8}{27}\text{ m}}$

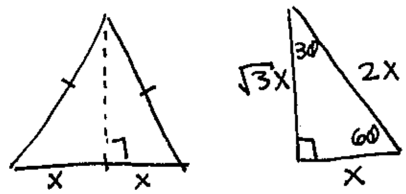
DISPLACEMENT:

$\int_0^1 v(t)\,dt = \frac{4}{27} + \left(-\frac{4}{27}\right) =$ $\boxed{\text{DISPLACEMENT: } 0\text{ m}}$

7. A solid has a circular base with equation $x^2 + y^2 = 4$. Parallel cross-sections perpendicular to the y-axis are equilateral triangles. Find the volume.







$b = 2x$

$h = \sqrt{3}x$

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(2x)(\sqrt{3}x)$$

$$A = \sqrt{3}x^2$$

$$\Delta V = \sqrt{3}x^2 \Delta y$$

$$\Delta V = \sqrt{3}(\sqrt{4 - y^2})^2 \Delta y$$

$$\Delta V = \sqrt{3}(4 - y^2)\Delta y$$

$$\Delta V = (4\sqrt{3} - \sqrt{3}y^2)\Delta y$$

$$V = 2\int_0^2 (4\sqrt{3} - \sqrt{3}y^2)\,dy$$

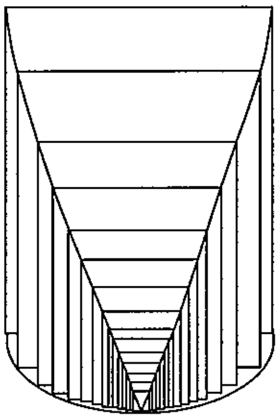
$$V = 2\left[4\sqrt{3}y - \frac{\sqrt{3}}{3}y^3 \Big|_0^2\right]$$

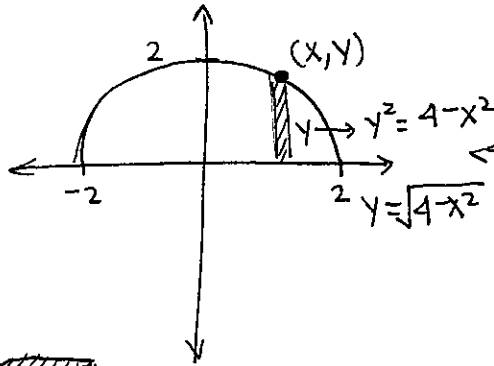
$$V = 2\left[4\sqrt{3}(2) - \frac{\sqrt{3}}{3}(8) - 0\right]$$

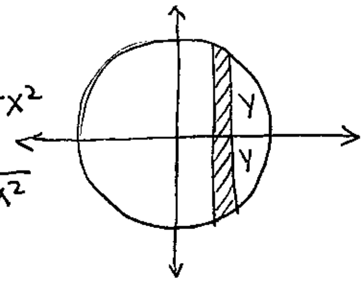
$$V = 16\sqrt{3} - \frac{16\sqrt{3}}{3}$$

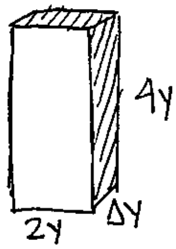
$$V = \frac{32\sqrt{3}}{3}$$

8. A solid has a circular base with equation $x^2 + y^2 = 4$. Parallel cross-sections perpendicular to the x-axis are rectangles **whose height is twice the length of its width**. Find the volume.









$x^2 + y^2 = 4$

$4 - x^2 = y^2$

$V = l \cdot w \cdot h$

$V = 2y \cdot \Delta x \cdot 4y$

$V = 8y^2 \Delta x$

$V = 8(4 - x^2)\Delta x$

$V = 8 \cdot 2 \int_0^2 (4 - x^2)\,dx$

$V = 16\left[4x - \frac{x^3}{3}\Big|_0^2\right]$

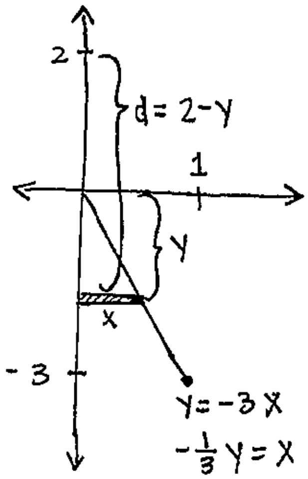
$V = 16\left[8 - \frac{8}{3} - 0\right]$

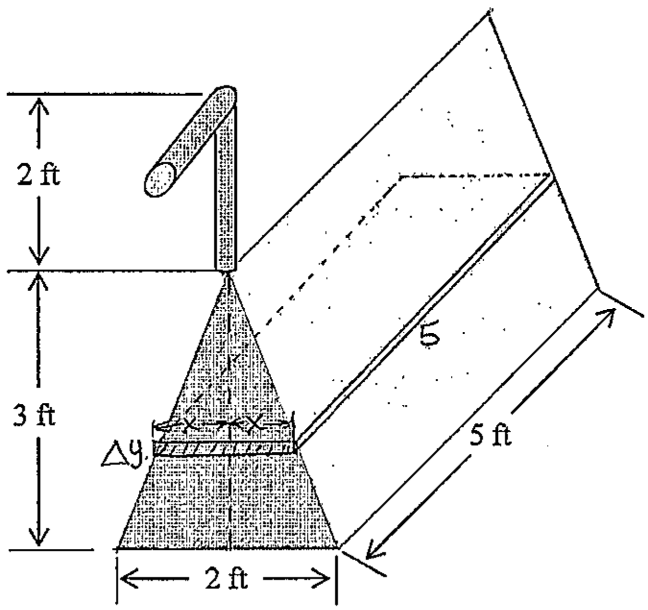
$V = 16\left[\frac{24}{3} - \frac{8}{3}\right]$

$V = 16\left(\frac{16}{3}\right)$

$\boxed{V = \frac{256}{3}}$

9. The tank below is full of heating oil with density 100 lb/ft^3. Find the work required to pump the oil out the outlet. See figure.





$V = lwh$

$V = 5 \cdot 2x \cdot \Delta Y$

$V = 10 x \Delta Y$

$V = -\frac{10}{3} y \Delta Y$ ft^3

WEIGHT $= \rho \cdot$ Volume

$= 100\left(-\frac{10}{3}\right) y \, \Delta Y$

$= -\frac{1000}{3} y \, \Delta Y$ lbs

WORK $= \int \rho \cdot$ volume $\cdot$ distance

$= -\frac{1000}{3} \int_{-3}^{0} y(2-y)\, dy$

$= -\frac{1000}{3} \int_{-3}^{0} 2y - y^2 \, dy$

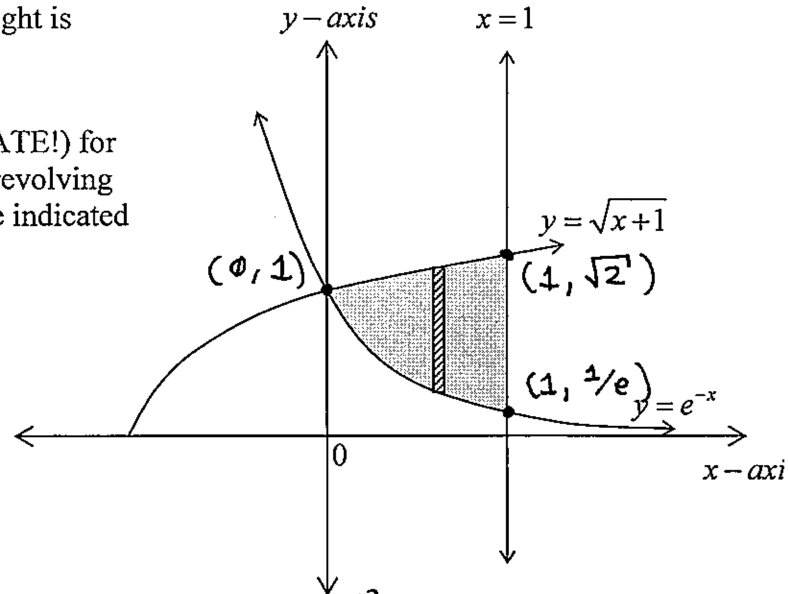
$= -\frac{1000}{3} \left[y^2 - \frac{y^3}{3} \Big|_{-3}^{0} \right]$

$= -\frac{1000}{3} \left[0 - \left(9 + \frac{27}{3} \right) \right]$

$= -\frac{1000}{3} (-18)$

$= \boxed{6000 \text{ ft} \cdot \text{lbs}}$

10. The region shown in the figure to the right is bounded by the graphs;
$y = e^{-x}$, $y = \sqrt{x+1}$, $x = 0$ and $x = 1$
Write the integral (DO NOT INTEGRATE!) for the volume V of the solid obtained by revolving the region about the stated line. Use the indicated method.



a) $y = -1$; Washer Method

$$\boxed{V = \pi \int_0^1 \Big[\big(\overbrace{1 + \sqrt{x+1}}^{R^2}\big)^2 - \big(\overbrace{1 + e^{-x}}^{r^2}\big)^2\Big]\,dx}$$

b) $x = -1$; Shell Method

$$\boxed{V = 2\pi \int_0^1 \overbrace{(1 + x)}^{r}\overbrace{(\sqrt{x+1} - e^{-x})}^{\ell}\,dx}$$

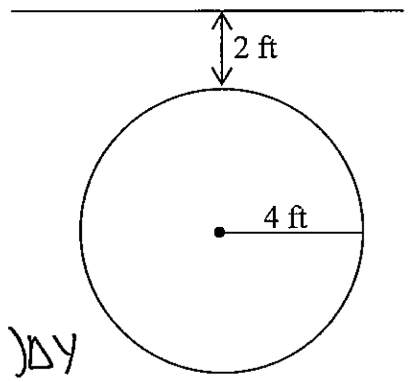
c) $y = 3$; Washer Method

$$\boxed{V = \pi \int_0^1 \Big[\big(\overbrace{3 - e^{-x}}^{R^2}\big)^2 - \big(\overbrace{3 - \sqrt{x+1}}^{r^2}\big)^2\Big]\,dx}$$

d) $x = 2$; Shell Method

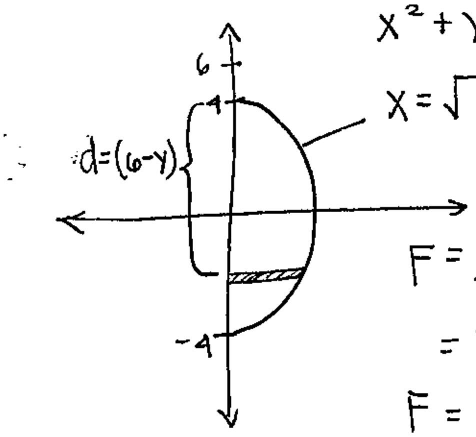
$$\boxed{V = 2\pi \int_0^1 \overbrace{(2 - x)}^{r}\overbrace{(\sqrt{x+1} - e^{-x})}^{\ell}\,dx}$$

11. The flat surface shown is submerged in a fluid. Find the fluid force against the surface. The fluid density is $10\dfrac{lbs}{ft^3}$. **Note:** $F=\int_a^b \rho h(x)w(x)\,dx$



$x^2+y^2=16$

$x=\sqrt{16-y^2}$



$F=\rho h(y)\,w(y)\Delta y$

$=10(6-y)(2x)\Delta y$

$F=10(6-y)(2\sqrt{16-y^2})\Delta y$

$$F=20\int_{-4}^{4}(6-y)(\sqrt{16-y^2})\,dy$$

$$F=20\int_{-4}^{4}6\sqrt{16-y^2}\,dy-\cancel{20\int_{-4}^{4}y\sqrt{16-y^2}\,dy}\to 0$$

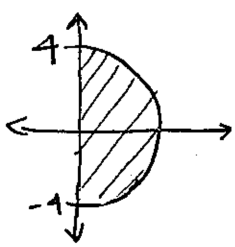
$u=16-y^2$

$du=-2y\,dy$

$-\frac{1}{2}du=y\,dy$

$y=4\to u=0$

$y=-4\to u=0$



$$F=120\int_{-4}^{4}\sqrt{16-y^2}\,dy$$

$=120(8\pi)$

$\boxed{F=960\pi\ \text{lbs}}$

$\frac{A}{2}=\frac{\pi r^2}{2}$

$=\frac{\pi\cdot 4^2}{2}$

$\frac{A}{2}=8\pi$

Math3A

Exam #03 Solutions

Fall 2016

1. Integrate the following. Rationalize the denominator if necessary and write your solution as a single fraction.

a) $\int_{1}^{2} \frac{6x-3x^2}{\sqrt{3x^2-x^3}}\,dx$

b) $\int_{6}^{9} \frac{x-3}{\sqrt{x-5}}\,dx$

2. Evaluate the integral by making the appropriate substitutions. Write your solution as a single fraction.

a) $\displaystyle\int_{\frac{9}{\pi}}^{\frac{18}{\pi}} \frac{\sec^2\left(\frac{3}{x}\right)}{x^2}\,dx$

b) $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \sin^2 3x \cos 3x \; dx$

3. Divide the specified interval into $n = 4$ subintervals of equal length and then compute $\sum_{k=1}^{4} f\left(x_k^*\right)\Delta x$ with x_k^* as the **midpoint** of the subinterval. **Write your solution as a single fraction.**

$$f(x) = \cos x \qquad \left[0, \frac{4\pi}{3}\right]$$

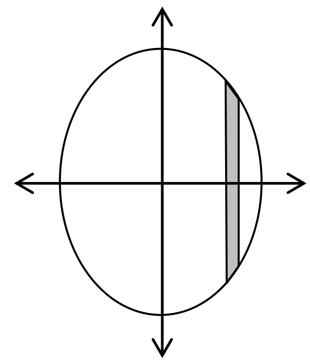
4. Find the exact arc length of the curve $y = 3x^{3/2} - 1$ from $x = 0$ to $x = 1$. Write your solutions as a single fraction. $L = \int_a^b \sqrt{1 + \left[f'(x)\right]^2}\, dx$

5. A particle moves with acceleration $a(t)\ m/s^2$ along an s-axis and has velocity $v_0\ \ m/s$ at time $t = 0$.

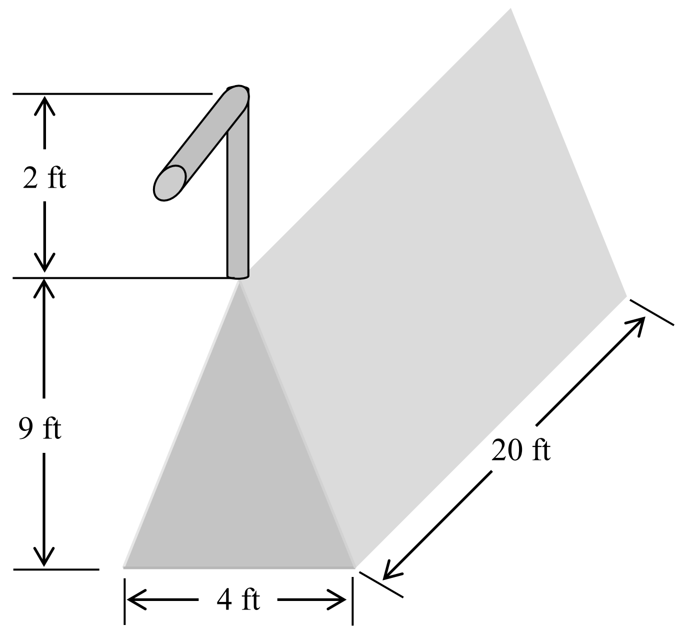
Find the displacement and the distance traveled by the particle during the given time interval.

$a(t) = -8t + 12$; $v(0) = -8$; $-2 \le t \le 2$

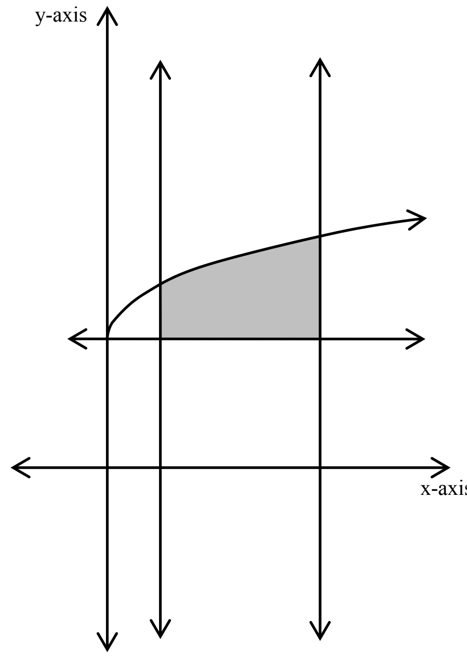
6. A solid has an elliptical base with equation $\frac{x^2}{4}+\frac{y^2}{9}=1$. Parallel cross-sections perpendicular to the x-axis are squares. Find the volume.



7. The tank below is filled to a depth of 6 feet with heating oil of density 10 lb/ft^3. Find the work required to pump the oil out the outlet. See figure.



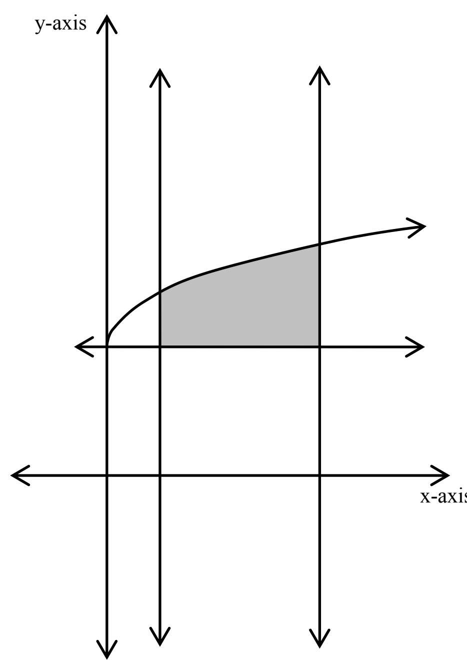
8. The region shown in the figure to the right is bounded by the graphs;
$y = \sqrt{x} + 3$, $y = 3$, $x = 1$ and $x = 4$
Write the integral for the volume V of the solid obtained by revolving the region about the stated axis **(DO NOT INTEGRATE!)**.



a) x-axis; Washer Method

b) y-axis; Washer Method

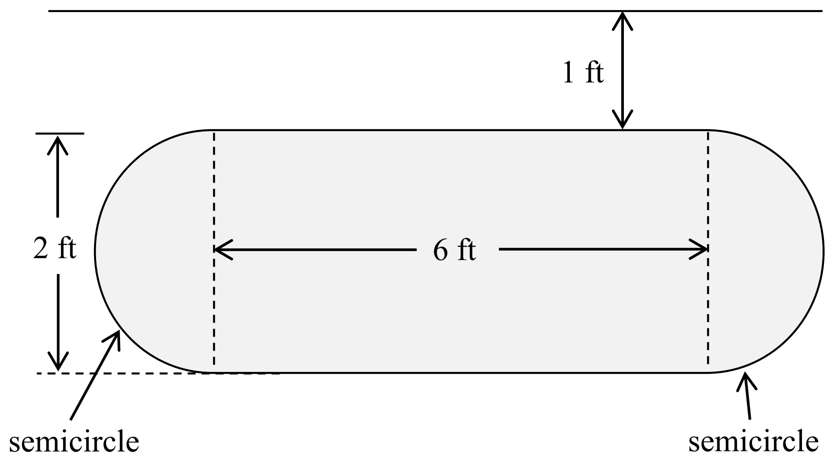
c) $y = -1$; Shell Method



d) $x = 5$; Shell Method

9. The flat surface shown is submerged vertically in a fluid. It is a rectangle with two semicircular ends. Find the fluid force against the surface. The fluid density is $10\dfrac{lbs}{ft^3}$.

Note: $F = \int_a^b \rho h(y) w(y)\, dy$



Math3A

Exam #03 Solutions

Fall 2016

1. Integrate the following. Rationalize the denominator if necessary and write your solution as a single fraction.

a) $\int_1^2 \frac{6x-3x^2}{\sqrt{3x^2-x^3}}\,dx$

$u = 3x^2 - x^3$

$du = 6x - 3x^2\,dx$

$x=2 \Rightarrow u = 3(2)^2 - 2^3$

$u = 12 - 8$

$u = 4$

$x = 1 \Rightarrow u = 3(1)^2 - 1^3$

$u = 3 - 1$

$u = 2$

$$\int_2^4 \frac{1}{\sqrt{u}}\,du$$

$$\int_2^4 u^{-\frac{1}{2}}\,du$$

$$2u^{\frac{1}{2}}\Big|_2^4$$

$$2[4^{1/2} - 2^{1/2}]$$

$$\boxed{2[2-\sqrt{2}]}$$

-or-

$$\boxed{4 - 2\sqrt{2}}$$

b) $\int_6^9 \frac{x-3}{\sqrt{x-5}}\,dx$

$u = x - 5 \Rightarrow x = u + 5$

$du = dx$

$x = 9 \Rightarrow u = 9 - 5 = 4$

$x = 6 \Rightarrow u = 6 - 5 = 1$

$$\int_1^4 \frac{u+5-3}{\sqrt{u}}\,du$$

$$\int_1^4 \frac{u+2}{\sqrt{u}}\,du$$

$$\int_1^4 \left[u^{\frac{1}{2}} + 2u^{-\frac{1}{2}}\right] du$$

$$\frac{2}{3}u^{\frac{3}{2}} + 4u^{\frac{1}{2}}\Big|_1^4$$

$$\left[\frac{2}{3}(8) + 4(2)\right] - \left[\frac{2}{3} + 4\right]$$

$$\frac{16}{3} + \frac{24}{3} - \frac{2}{3} - \frac{12}{3}$$

$$\boxed{\frac{26}{3}}$$

2. Evaluate the integral by making the appropriate substitutions. Write your solution as a single fraction.

a) $\displaystyle\int_{\frac{9}{\pi}}^{\frac{18}{\pi}} \frac{\sec^2\left(\frac{3}{x}\right)}{x^2}\,dx$

$u = \frac{3}{x}$

$du = -\frac{3}{x^2}\,dx$

$-\frac{1}{3}du = \frac{1}{x^2}\,dx$

$x = \frac{18}{\pi} \Rightarrow u = \frac{3}{18/\pi} = \frac{\pi}{6}$

$x = \frac{9}{\pi} \Rightarrow u = \frac{3}{9/\pi} = \frac{\pi}{3}$

$-\frac{1}{3}\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \sec^2 u\,du$

$\frac{1}{3}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 u\,du$

$\frac{1}{3}\tan u\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$\frac{1}{3}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{6}\right]$

$\frac{1}{3}\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right]$

$\frac{1}{3}\left[\frac{3\sqrt{3}}{3} - \frac{\sqrt{3}}{3}\right]$

$\frac{1}{3}\left[\frac{2\sqrt{3}}{3}\right]$

$\boxed{\frac{2\sqrt{3}}{9}}$

b) $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \sin^2 3x\cos 3x\,dx$

$u = \sin 3x$

$du = 3\cos 3x\,dx$

$\frac{1}{3}du = \cos 3x\,dx$

$x = \frac{\pi}{3} \Rightarrow u = \sin\left[3\left(\frac{\pi}{6}\right)\right] = \sin\frac{\pi}{2} = 1$

$x = \frac{\pi}{12} \Rightarrow u = \sin\left[3\left(\frac{\pi}{12}\right)\right] = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\frac{1}{3}\int_{\frac{\sqrt{2}}{2}}^{1} u^2\,du$

$\frac{1}{3}\cdot\frac{u^3}{3}\Big|_{\frac{\sqrt{2}}{2}}^{1}$

$\frac{1}{9}\left[1^3 - \left(\frac{\sqrt{2}}{2}\right)^3\right]$

$\frac{1}{9}\left[1 - \frac{2\sqrt{2}}{8}\right]$

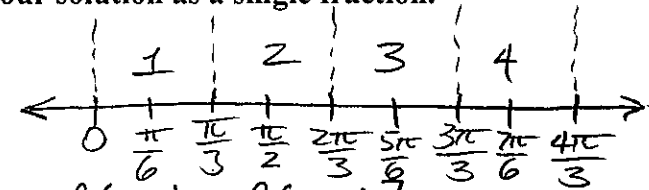
$\frac{1}{9}\left[\frac{4}{4} - \frac{\sqrt{2}}{4}\right]$

$\frac{1}{9}\left[\frac{4-\sqrt{2}}{4}\right]$

$\boxed{\frac{4-\sqrt{2}}{36}}$

3. Divide the specified interval into $n = 4$ subintervals of equal length and then compute $\sum_{k=1}^{4} f\left(x_k^*\right)\Delta x$ with x_k^* as the **midpoint** of the subinterval. **Write your solution as a single fraction.**

$\Delta x = \frac{b-a}{n} = \frac{\frac{4\pi}{3}-0}{4} = \frac{\pi}{3}$ $\quad f(x) = \cos x \quad \left[0, \frac{4\pi}{3}\right]$



$$\sum_{k=1}^{4} f(x_k^*)\Delta x = \frac{\pi}{3}\left[f\left(\frac{\pi}{6}\right) + f\left(\frac{\pi}{2}\right) + f\left(\frac{5\pi}{6}\right) + f\left(\frac{7\pi}{6}\right)\right]$$

$$= \frac{\pi}{3}\left[\cos\frac{\pi}{6} + \cos\frac{\pi}{2} + \cos\frac{5\pi}{6} + \cos\frac{7\pi}{6}\right]$$

$$= \frac{\pi}{3}\left[\frac{\sqrt{3}}{2} + 0 + \left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{\pi}{3}\left[-\frac{\sqrt{3}}{2}\right]$$

$$= \boxed{-\frac{\sqrt{3}\,\pi}{6}}$$

4. Find the exact arc length of the curve $y = 3x^{3/2} - 1$ from $x = 0$ to $x = 1$. Write your solutions as a single fraction. $L = \int_a^b \sqrt{1 + \left[f'(x)\right]^2}\, dx$

$f(x) = 3x^{3/2} - 1$

$f'(x) = \frac{9}{2}x^{\frac{1}{2}} dx$

$L = \int_0^1 \sqrt{1 + \left(\frac{9}{2}x^{\frac{1}{2}}\right)^2}\, dx$

$= \int_0^1 \sqrt{1 + \frac{81}{4}x}\, dx$

$= \int_0^1 \sqrt{\frac{4 + 81x}{4}}\, dx$

$= \frac{1}{2}\int_0^1 \sqrt{4 + 81x}\, dx$

$u = 4 + 81x$

$du = 81\, dx$

$\frac{1}{81} du = dx$

$x = 1 \Rightarrow u = 4 + 81(1) = 85$

$x = 0 \Rightarrow u = 4 + 81(0) = 4$

$L = \frac{1}{2} \cdot \frac{1}{81}\int_4^{85} u^{\frac{1}{2}}\, du$

$= \frac{1}{162} \cdot \frac{2}{3}\left[u^{\frac{3}{2}}\Big|_4^{85}\right]$

$= \frac{1}{243}\left[85^{3/2} - 4^{3/2}\right]$

$= \boxed{\frac{85\sqrt{85} - 8}{243}}$

5. A particle moves with acceleration $a(t)\ m/s^2$ along an s-axis and has velocity $v_0\ m/s$ at time $t = 0$. Find the displacement and the distance traveled by the particle during the given time interval.

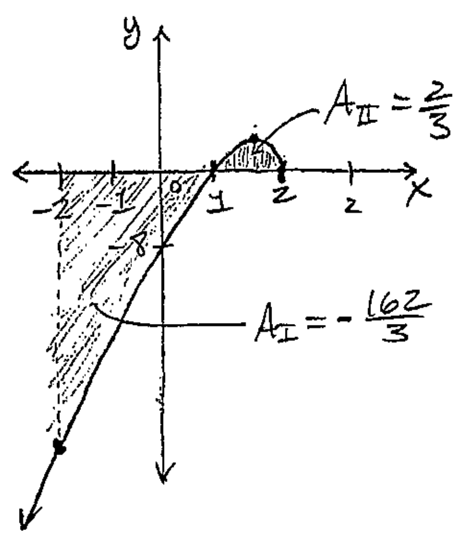
$a(t) = -8t + 12$; $v(0) = -8$; $-2 \le t \le 2$

$$V(t) = \int a(t)\,dt$$
$$= \int (-8t+12)\,dt$$
$$V(t) = -4t^2 + 12t + C$$

Using $V(0) = -8$,

$$-8 = -4(0)^2 + 12(0) + C$$
$$-8 = C$$

$$V(t) = -4t^2 + 12t - 8$$
$$V(t) = -4(t^2 - 3t + 2)$$
$$V(t) = -4(t-2)(t-1)$$



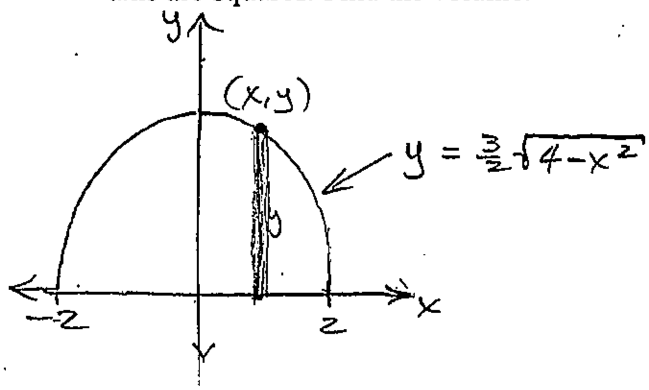
$$\boxed{\begin{array}{l} \int_{-2}^{1} (-4t^2+12t-8)\,dt = -\frac{162}{3} \\ \int_{1}^{2} (-4t^2+12t-8)\,dt = \frac{2}{3} \end{array}}$$

Displacement:

$$\text{Disp.} = \int_a^b V(t)\,dt$$
$$= \int_{-2}^{2} (-4t^2 + 12t - 8)\,dt$$
$$= -\frac{4}{3}t^3 + 6t^2 - 8t \Big|_{-2}^{2}$$
$$= -\frac{4}{3}(8) + 6(4) - 8(2) - \left(-\frac{4}{3}(-8) + 6(4) - 8(-2)\right)$$
$$= -\frac{32}{3} + \cancel{24} - 16 - \frac{32}{3} - \cancel{24} - 16$$
$$= -\frac{64}{3} - 32$$
$$= -\frac{64}{3} - \frac{96}{3}$$
$$= \boxed{-\frac{160}{3}\ m}$$

$$\text{Distance} = \int_{-2}^{2} |V(t)|\,dt$$
$$= \int_{-2}^{1} (4t^2 - 12t + 8)\,dt + \int_{1}^{2} (-4t^2 + 12t - 8)\,dt$$
$$= \frac{4}{3}t^3 - 6t^2 + 8t \Big|_{-2}^{1} + -\frac{4}{3}t^3 + 6t^2 - 8t \Big|_{1}^{2}$$
$$= \frac{4}{3} - 6 + 8 - \left(-\frac{32}{3} - 24 - 16\right) - \frac{32}{3} + 24 - 16 - \left(-\frac{4}{3} + 6 - 8\right)$$
$$= \frac{4}{3} + 2 + \cancel{\frac{32}{3}} + 24 + 16 - \cancel{\frac{32}{3}} + 8 + \frac{4}{3} - 6 + 8$$
$$= \frac{8}{3} + 58 - 6$$
$$= \frac{8}{3} + 52$$
$$= \frac{8}{3} + \frac{156}{3}$$
$$= \boxed{\frac{164}{3}\ m}$$

6. A solid has an elliptical base with equation $\frac{x^2}{4}+\frac{y^2}{9}=1$. Parallel cross-sections perpendicular to the x-axis are squares. Find the volume.



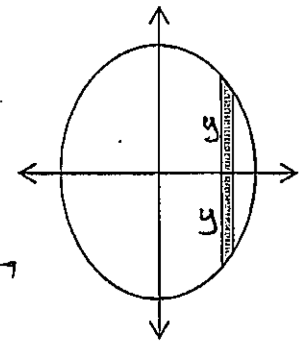
$$\frac{y^2}{9} = 1 - \frac{x^2}{4}$$

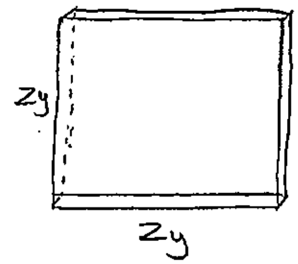
$$y^2 = 9 - \frac{9}{4}x^2$$

$$y = \sqrt{9 - \frac{9}{4}x^2}$$

$$y = \sqrt{\frac{9}{4}(4-x^2)}$$

$$y = \frac{3}{2}\sqrt{4-x^2}$$





$$V_{slice} = s^2 \Delta x$$

$$= (2y)^2 \Delta x$$

$$= 4y^2 \Delta x$$

$$= 4\left(\frac{9}{4}\right)(4-x^2)\Delta x$$

$$= 9(4-x^2)\Delta x$$

$$V = \int_{-2}^{2} 9(4-x^2)\,dx$$

$$= 18\int_0^2 (4-x^2)\,dx$$

$$= 18\left[4x - \frac{x^3}{3}\Big|_0^2\right]$$

$$= 18\left[8 - \frac{8}{3} - 0\right]$$

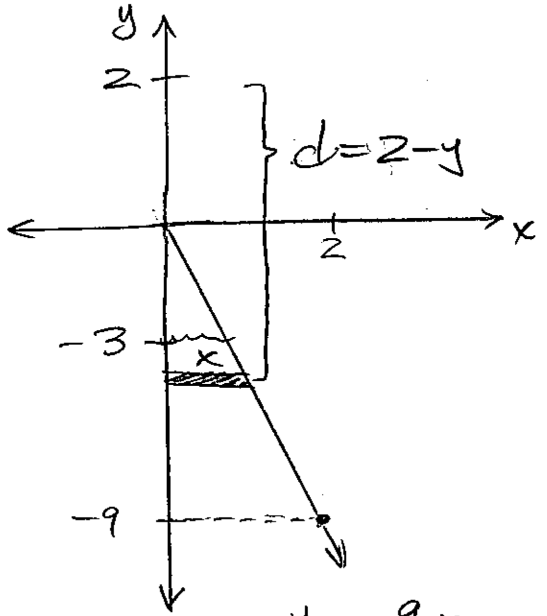
$$= 18\left[\frac{24-8}{3}\right]$$

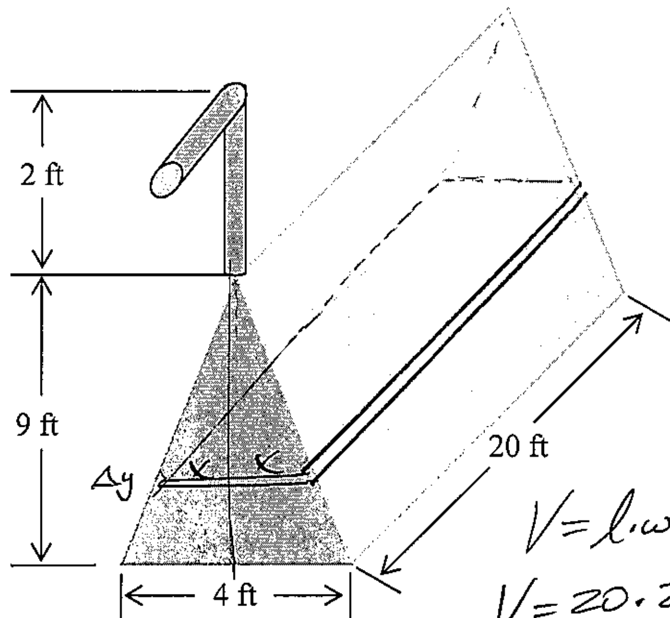
$$= \frac{\cancel{18}^{\,6}}{1}\cdot\frac{16}{\cancel{3}}$$

$$= 6 \cdot 16$$

$$= \boxed{96}$$

7. The tank below is filled to a depth of 6 feet with heating oil of density 10 lb/ft^3. Find the work required to pump the oil out the outlet. See figure.





$y = -\frac{9}{2}x$

$-\frac{2}{9}y = x$

$V = l \cdot w \cdot h$

$V = 20 \cdot 2x \cdot \Delta y$

$V = 40x \, \Delta y$

$V = 40\left(-\frac{2}{9}y\right)\Delta y$

$V = -\frac{80}{9}y \, \Delta y$

$$W = \int_a^b F(y)\, dy$$

$$= \int_{-9}^{-3} 10\left(-\frac{80}{9}y\right)(2-y)\, dy$$

$$= \frac{800}{9}\int_{-9}^{-3} y(y-2)\, dy$$

$$= \frac{800}{9}\int_{-9}^{-3} (y^2-2y)\, dy$$

$$= \frac{800}{9}\left[\frac{y^3}{3} - y^2 \Big|_{-9}^{-3}\right]$$

$$= \frac{800}{9}\left[(-9-9)-(-243-81)\right]$$

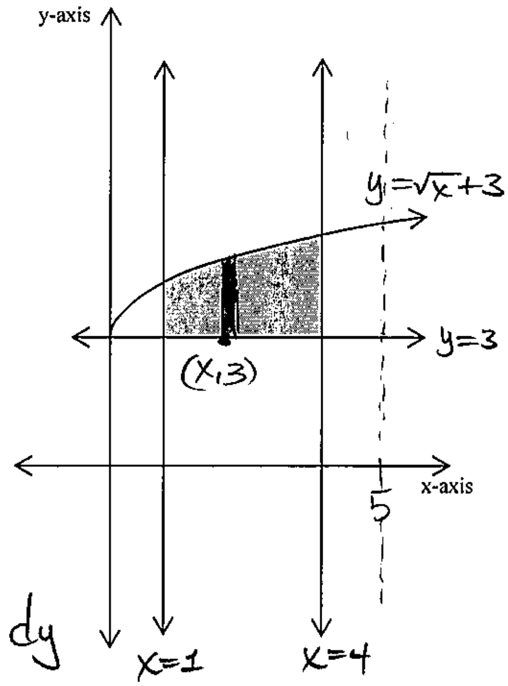
$$= \frac{800}{9}\left[-18+243+81\right]$$

$$= \frac{800}{9}(306)$$

$$= 800(34)$$

$$= \boxed{27{,}200 \text{ ft} \cdot \text{lbs}}$$

8. The region shown in the figure to the right is bounded by the graphs;
$y=\sqrt{x}+3$, $y=3$, $x=1$ and $x=4$
Write the integral for the volume V of the solid obtained by revolving the region about the stated axis **(DO NOT INTEGRATE!)**.



a) x-axis; Washer Method

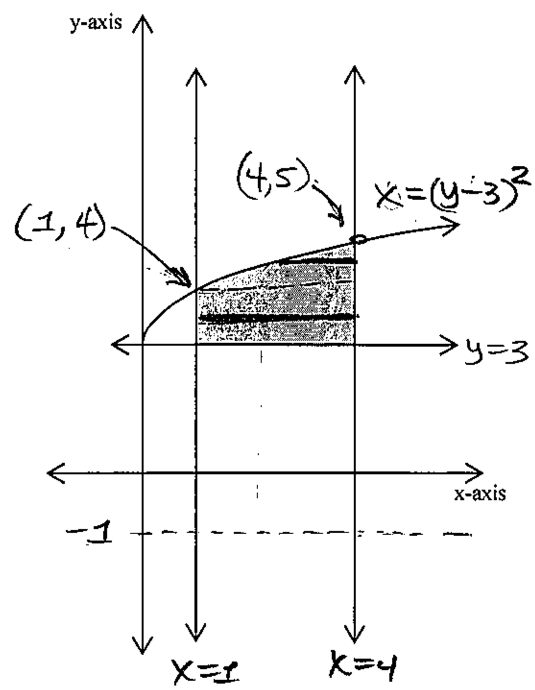
$$V=\pi\int_1^4\left[(\sqrt{x}+3)^2-(3)^2\right]dx$$

b) y-axis; Washer Method

$$V=\pi\int_3^4\left[4^2-1^2\right]dy+\pi\int_4^5\left\{4^2-\left[(y-3)^2\right]^2\right\}dy$$

c) $y=-1$; Shell Method

$$V=2\pi\int_3^4(y+1)(4-1)\,dy+2\pi\int_4^5(y+1)\left[4-(y-3)^2\right]dy$$



d) $x=5$; Shell Method

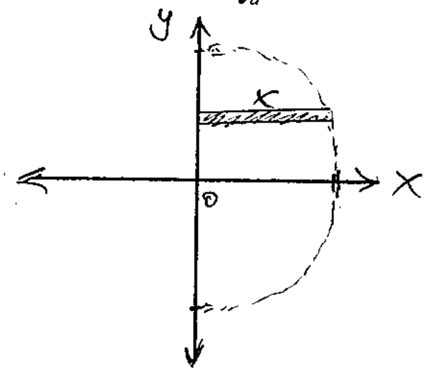
$$V=2\pi\int_1^4(5-x)(\sqrt{x}+3-3)\,dx$$

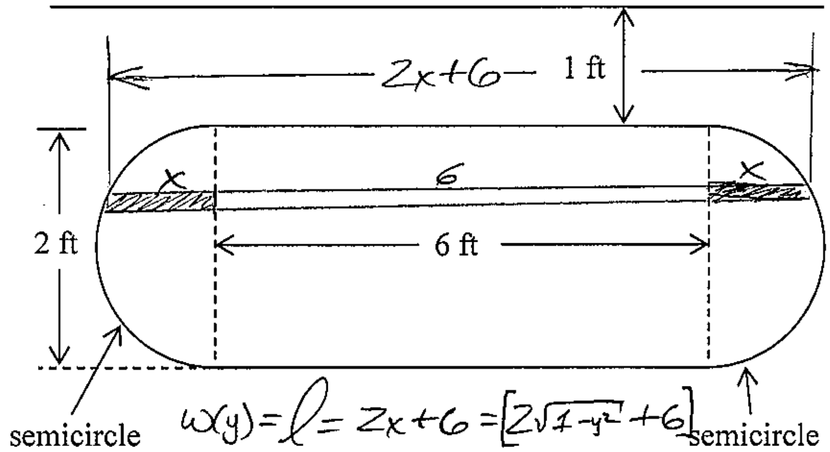
-or-

$$V=2\pi\int_1^4(5-x)\sqrt{x}\,dx$$

9. The flat surface shown is submerged vertically in a fluid. It is a rectangle with two semicircular ends. Find the fluid force against the surface. The fluid density is $10\frac{lbs}{ft^3}$.

Note: $F=\int_a^b \rho h(y)w(y)\,dy$





$w(y)=\ell=2x+6=\left[2\sqrt{1-y^2}+6\right]$

$x^2+y^2=1$

$x^2=1-y^2$

$x=\sqrt{1-y^2}$

$$F=\int_a^b \rho\, h(y)\, w(y)\, dy$$

$$F=\int_{-1}^{1}\left[10\cdot(2-y)\left(2\sqrt{1-y^2}+6\right)\right]dy$$

$$F=20\int_{-1}^{1}\left[(2-y)\left(\sqrt{1-y^2}+3\right)\right]dy$$

$$F=20\int_{-1}^{1}\left[\underbrace{2\sqrt{1-y^2}}_{\substack{A=\frac{1}{2}(\pi)(1)^2\\ A=\frac{\pi}{2}}}+6\underbrace{-y\sqrt{1-y^2}}_{\text{Odd Function}}-3y\right]dy$$

$$F=20\left[2\left(\frac{\pi}{2}\right)+6y\Big|_{-1}^{1}-0-\frac{3y^2}{2}\Big|_{-1}^{1}\right]$$

$$F=20\left[\pi+(6-(-6))-\left(\frac{3}{2}-\frac{3}{2}\right)\right]$$

$$F=20\left[\pi+12\right]$$

$$\boxed{F=20\pi+240 \text{ lb}}$$

Math3A

Exam #03 Solutions

1. Integrate the following. Rationalize the denominator if necessary and write your solution as a single fraction.

   a) $\int_{\pi/6}^{2\pi/3} \frac{\cot x}{\sin x}\, dx$

   b) $\int_{0}^{\pi/3} \sec^3 x \tan x \, dx$

2. Evaluate the integral by making the appropriate substitutions. Write your solution as a single fraction.

a) $\displaystyle\int_{\frac{9}{\pi}}^{\frac{18}{\pi}} \frac{\sec^2\left(\frac{3}{x}\right)}{x^2}\,dx$

b) $\displaystyle\int_{-1}^{1/2} \frac{2x-5}{\sqrt{2x+3}}\,dx$

3. Given, $\int_{-2}^{3} f(x)\,dx = -2$, $\int_{-2}^{3} g(x)\,dx = -4$, $\frac{2}{3}\int_{-2}^{3} h(x)\,dx = 1$, find the following:

a) $2\int_{-2}^{3}[g(x) - 2x]\,dx$

b) $\int_{-2}^{3}\left[2f(x) - \frac{3}{2}h(x)\right]dx$

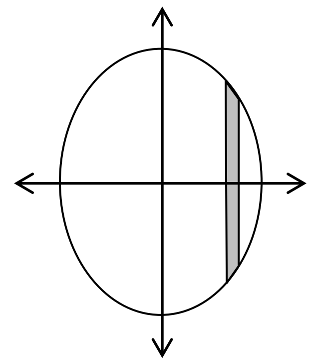
4. A particle moves along the *s*-axis. Use the given information to find the position function of the particle.

$$a(t) = 4 - \sin 3t\ ; \quad v\left(\frac{\pi}{6}\right) = 0; \quad s(0) = \pi$$

5. A particle moves with acceleration $a(t)\ m/s^2$ along an s-axis and has velocity $v_0\ \ m/s$ at time $t=0$. Find the displacement and the distance traveled by the particle during the given time interval.

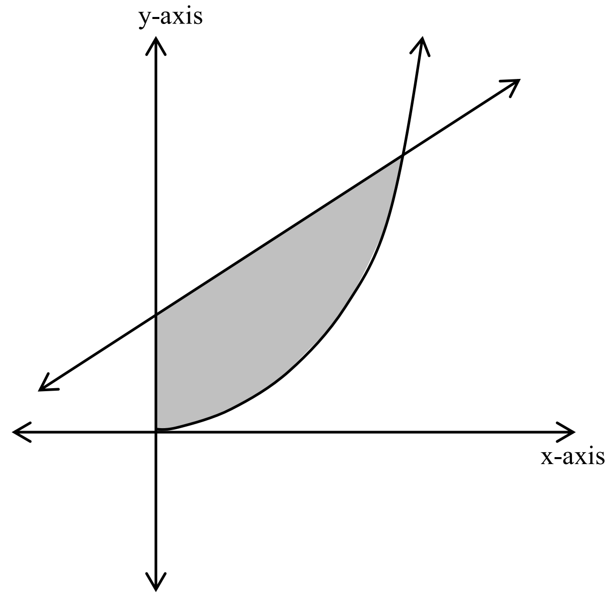
$a(t)=-8t+12$; $v(0)=-8$; $-2\le t\le 2$

6. A solid has an elliptical base with equation $\frac{x^2}{4}+\frac{y^2}{9}=1$. Parallel cross-sections perpendicular to the x-axis are squares. Find the volume.

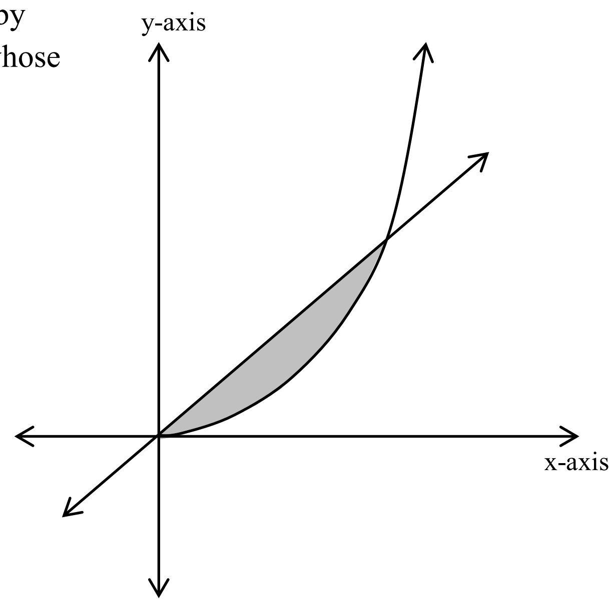


7. Answer the following questions.

a) The region shown in the figure to the right is bounded by $y = x^2$, $y = x + 2$ and $x = 0$. Find the area of the bounded region.



b) The region shown in the figure to the right is bounded by $y = x^2$, $y = x$ and $x = 0$. Find the volume of the solid whose base is the shaded region where parallel cross-sections perpendicular to the x-axis are semi-circles.



8. Find the average value of the function $f(x)=\sqrt{3x}$ on the interval $[4,9]$.

9. Divide the specified interval into $n=4$ subintervals of equal length and then compute $\sum_{k=1}^{4} f(x_k^*)\Delta x$ with x_k^* as the left endpoint of the subinterval. **Write your solution as a single fraction.**

$$f(x)=\cos x \quad \left[0,\frac{2\pi}{3}\right]$$

Math3A

Exam #03

Summer 2016 Solution

1. Integrate the following. Rationalize the denominator if necessary and write your solution as a single fraction.

a) $\int_{\pi/6}^{2\pi/3} \frac{\cot x}{\sin x}\, dx$

$$\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\, dx$$

$$\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \frac{\cos x}{\sin^2 x}\, dx$$

$u = \sin x$

$du = \cos x \, dx$

$x = \frac{2\pi}{3} \Rightarrow u = \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6} \Rightarrow u = \sin \frac{\pi}{6} = \frac{1}{2}$

$$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{u^2}\, du$$

$$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} u^{-2}\, du$$

$$-\frac{1}{u}\Big|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$\left(-\frac{2}{\sqrt{3}}\right) - (-2)$$

$$-\frac{2\sqrt{3}}{3} + 2$$

$$\boxed{\frac{-2\sqrt{3}+6}{3}}$$

b) $\int_0^{\pi/3} \sec^3 x \tan x \, dx$

$$\int_0^{\frac{\pi}{3}} \sec^2 x \cdot \sec x \tan x \, dx$$

$u = \sec x$ $\qquad x = \frac{\pi}{3} \Rightarrow u = \sec \frac{\pi}{3} = 2$

$du = \sec x \tan x \, dx$ $\qquad x = 0 \Rightarrow u = \sec 0 = 1$

$$\int_1^2 u^2\, du$$

$$\frac{u^3}{3}\Big|_1^2$$

$$\frac{8}{3} - \frac{1}{3}$$

$$\boxed{\frac{7}{3}}$$

2. Evaluate the integral by making the appropriate substitutions. Write your solution as a single fraction.

a) $\displaystyle\int_{\frac{9}{\pi}}^{\frac{18}{\pi}} \frac{\sec^2\left(\frac{3}{x}\right)}{x^2}\,dx$

$u = \frac{3}{x}$

$du = -\frac{3}{x^2}\,dx$

$-\frac{1}{3}du = \frac{1}{x^2}dx$

$x = \frac{18}{\pi} \Rightarrow u = \frac{3}{\frac{18}{\pi}}$

$u = \frac{3\pi}{18}$

$u = \frac{\pi}{6}$

$x = \frac{9}{\pi} \Rightarrow u = \frac{3}{\frac{9}{\pi}}$

$u = \frac{3\pi}{9}$

$u = \frac{\pi}{3}$

$-\frac{1}{3}\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \sec^2 u\,du$

$\frac{1}{3}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 u\,du$

$\frac{1}{3}\tan u\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$\frac{1}{3}\left[\tan\frac{\pi}{3} - \tan\frac{\pi}{6}\right]$

$\frac{1}{3}\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right]$

$\frac{1}{3}\left[\frac{3\sqrt{3}}{3} - \frac{\sqrt{3}}{3}\right]$

$\frac{1}{3}\left[\frac{2\sqrt{3}}{3}\right]$

$\boxed{\frac{2\sqrt{3}}{9}}$

b) $\displaystyle\int_{-1}^{1/2} \frac{2x-5}{\sqrt{2x+3}}\,dx$

$u = 2x+3$

$du = 2dx$

$\frac{1}{2}du = dx$

$2x+3 = u$

$2x = u-3$

$x = \frac{1}{2} \Rightarrow u = 2(\frac{1}{2})+3 = 4$

$x = -1 \Rightarrow u = 2(-1)+3 = 1$

$\frac{1}{2}\int_1^4 \left[\frac{u-3-5}{u^{1/2}}\right]du$

$\frac{1}{2}\int_1^4 \left[\frac{u-8}{u^{1/2}}\right]du$

$\frac{1}{2}\int_1^4 \left[u^{\frac{1}{2}} - 8u^{-\frac{1}{2}}\right]du$

$\frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}} - 16u^{\frac{1}{2}}\Big|_1^4\right]$

$\frac{1}{2}\left[\left(\frac{2}{3}\cdot 8 - 16\cdot 2\right) - \left(\frac{2}{3} - 16\right)\right]$

$\frac{1}{2}\left[\frac{16}{3} - \frac{96}{3} - \frac{2}{3} + \frac{48}{3}\right]$

$\frac{1}{2}\left[\frac{-34}{3}\right]$

$\boxed{-\frac{17}{3}}$

3. Given, $\int_{-2}^{3} f(x)\,dx = -2$, $\int_{-2}^{3} g(x)\,dx = -4$, $\frac{2}{3}\int_{-2}^{3} h(x)\,dx = 1$, find the following:

a) $2\int_{-2}^{3}[g(x)-2x]\,dx$

$2\int_{-2}^{3} g(x)\,dx - 2\int_{-2}^{3} 2x\,dx$

$2(-4) - 2\left[x^2\Big|_{-2}^{3}\right]$

$-8-2[9-4]$

$-8-2(5)$

$-8-10$

$\boxed{-18}$

b) $\int_{-2}^{3}\left[2f(x)-\frac{3}{2}h(x)\right]dx$

$2\int_{-2}^{3} f(x)\,dx - \frac{3}{2}\int_{-2}^{3} h(x)\,dx$

$2(-2) - \frac{3}{2}\left[\frac{3}{2}\right]$

$-4 - \frac{9}{4}$

$\frac{-16-9}{4}$

$\boxed{-\frac{25}{4}}$

4. A particle moves along the s-axis. Use the given information to find the position function of the particle.

$$a(t) = 4 - \sin t\,; \quad v\left(\frac{\pi}{2}\right) = 0; \quad s(0) = \pi$$

$V(t) = \int a(t)\,dt$

$V(t) = \int (4 - \sin t)\,dt$

$V(t) = 4t + \cos t + C$

Using $\left(\frac{\pi}{2}, 0\right)$,

$0 = 4\left(\frac{\pi}{2}\right) + \cancel{\cos\frac{\pi}{2}}^{\,0} + C$

$0 = 2\pi + C$

$-2\pi = C$

$V(t) = 4t + \cos t - 2\pi$

$S(t) = \int V(t)\,dx$

$S(t) = \int (4t + \cos t - 2\pi)\,dt$

$S(t) = 2t^2 + \sin t - 2\pi t + C$

Using $(0, \pi)$,

$\pi = 2(0)^2 + \sin 0 - 2\pi(0) + C$

$\pi = C$

$\boxed{S(t) = 2t^2 + \sin t - 2\pi t + \pi}$

5. A particle moves with acceleration $a(t)\ m/s^2$ along an s-axis and has velocity $v_0\ m/s$ at time $t = 0$. Find the displacement and the distance traveled by the particle during the given time interval.

$a(t) = -8t + 12$; $v(0) = -8$; $-2 \le t \le 2$

$V(t) = \int a(t)\,dt$

$= \int(-8t+12)\,dt$

$V(t) = -4t^2 + 12t + C$

Using $V(0) = -8$,

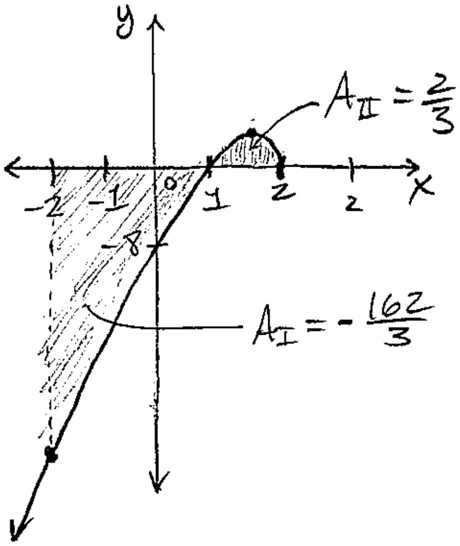
$-8 = -4(0)^2 + 12(0) + C$

$-8 = C$

$V(t) = -4t^2 + 12t - 8$

$V(t) = -4(t^2 - 3t + 2)$

$V(t) = -4(t-2)(t-1)$



$$\boxed{\begin{aligned}\int_{-2}^{1}(-4t^2+12t-8)\,dt &= -\frac{162}{3}\\ \int_{1}^{2}(-4t^2+12t-8)\,dt &= \frac{2}{3}\end{aligned}}$$

Displacement:

Disp. $= \int_a^b V(t)\,dt$

$= \int_{-2}^{2}(-4t^2+12t-8)\,dt$

$= -\frac{4}{3}t^3 + 6t^2 - 8t\Big|_{-2}^{2}$

$= -\frac{4}{3}(8) + 6(4) - 8(2) - \left(-\frac{4}{3}(-8) + 6(4) - 8(-2)\right)$

$= -\frac{32}{3} + \cancel{24} - 16 - \frac{32}{3} - \cancel{24} - 16$

$= -\frac{64}{3} - 32$

$= -\frac{64}{3} - \frac{96}{3}$

$= \boxed{-\frac{160}{3}\ m}$

Distance $= \int_{-2}^{2}|V(t)|\,dt$

$= \int_{-2}^{1}(4t^2 - 12t + 8)\,dt + \int_{1}^{2}(-4t^2 + 12t - 8)\,dt$

$= \frac{4}{3}t^3 - 6t^2 + 8t\Big|_{-2}^{1} + -\frac{4}{3}t^3 + 6t^2 - 8t\Big|_{1}^{2}$

$= \frac{4}{3} - 6 + 8 - \left(-\frac{32}{3} - 24 - 16\right) - \frac{32}{3} + 24 - 16 - \left(-\frac{4}{3} + 6 - 8\right)$

$= \frac{4}{3} + 2 + \cancel{\frac{32}{3}} + 24 + 16 - \cancel{\frac{32}{3}} + 8 + \frac{4}{3} - 6 + 8$

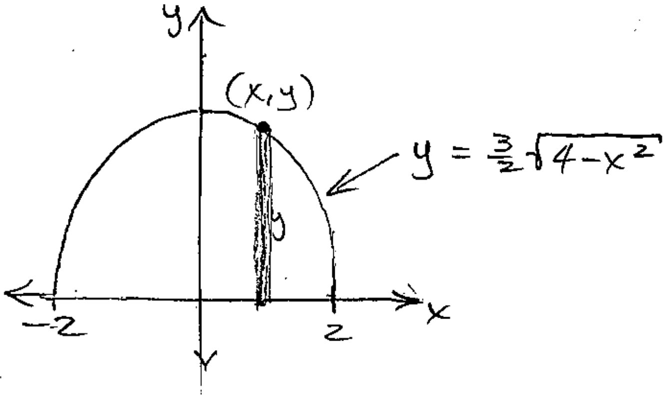
$= \frac{8}{3} + 58 - 6$

$= \frac{8}{3} + 52$

$= \frac{8}{3} + \frac{156}{3}$

$= \boxed{\frac{164}{3}\ m}$

6. A solid has an elliptical base with equation $\frac{x^2}{4}+\frac{y^2}{9}=1$. Parallel cross-sections perpendicular to the x-axis are squares. Find the volume.



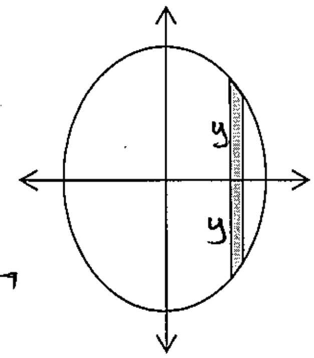
$$\frac{y^2}{9}=1-\frac{x^2}{4}$$

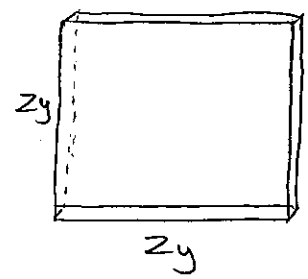
$$y^2=9-\frac{9}{4}x^2$$

$$y=\sqrt{9-\frac{9}{4}x^2}$$

$$y=\sqrt{\frac{9}{4}(4-x^2)}$$

$$y=\frac{3}{2}\sqrt{4-x^2}$$





$$V_{slice}=s^2\Delta x$$

$$=(2y)^2\Delta x$$

$$=4y^2\Delta x$$

$$=4\left(\frac{9}{4}\right)(4-x^2)\Delta x$$

$$=9(4-x^2)\Delta x$$

$$V=\int_{-2}^{2}9(4-x^2)\,dx$$

$$=18\int_0^2(4-x^2)\,dx$$

$$=18\left[4x-\frac{x^3}{3}\Big|_0^2\right]$$

$$=18\left[8-\frac{8}{3}-0\right]$$

$$=18\left[\frac{24-8}{3}\right]$$

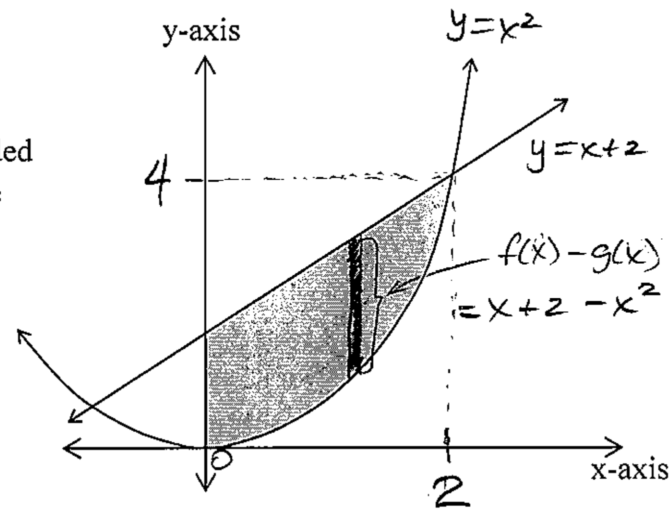
$$=\frac{\cancel{18}^{6}}{1}\cdot\frac{16}{\cancel{3}}$$

$$=6\cdot 16$$

$$=\boxed{96}$$

7. Answer the following questions.

a) The region shown in the figure to the right is bounded by $y = x^2$, $y = x + 2$ and $x = 0$. Find the area of the bounded region.



$$\int_0^2 [f(x) - g(x)]\,dx$$

$$\int_0^2 [x + 2 - x^2]\,dx$$

$$\frac{x^2}{2} + 2x - \frac{x^3}{3}\Big|_0^2$$

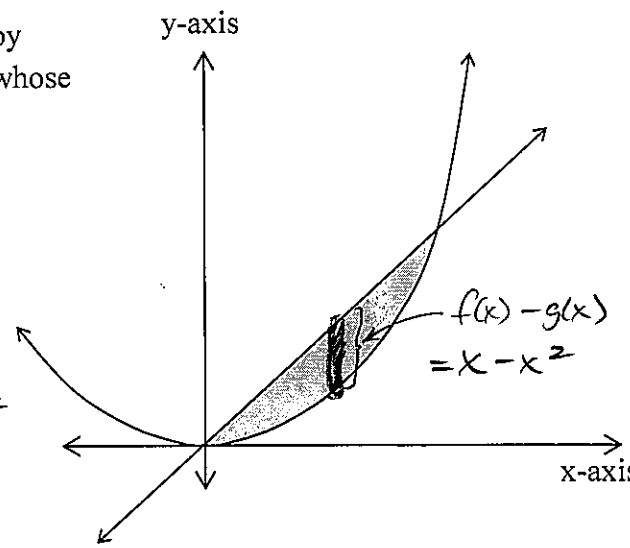
$$2 + 4 - \frac{8}{3} - (0)$$

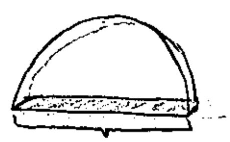
$$6 - \frac{8}{3}$$

$$\frac{18-8}{3}$$

$$\boxed{\frac{10}{3}}$$

b) The region shown in the figure to the right is bounded by $y = x^2$, $y = x$ and $x = 0$. Find the volume of the solid whose base is the shaded region where parallel cross-sections perpendicular to the x-axis are semi-circles.





$$V = \frac{\pi}{8}\int_0^1 (x^2 - 2x^3 + x^4)\,dx$$

$$= \frac{\pi}{8}\left[\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\Big|_0^1\right]$$

$$= \frac{\pi}{8}\left[\frac{1}{3} - \frac{1}{2} + \frac{1}{5} - 0\right]$$

$$= \frac{\pi}{8}\left[\frac{10-15+6}{30}\right]$$

$$= \frac{\pi}{8}\left[\frac{1}{30}\right]$$

$$= \boxed{\frac{\pi}{240}}$$

$D = x - x^2$

$r = \frac{1}{2}D = \frac{x-x^2}{2}$

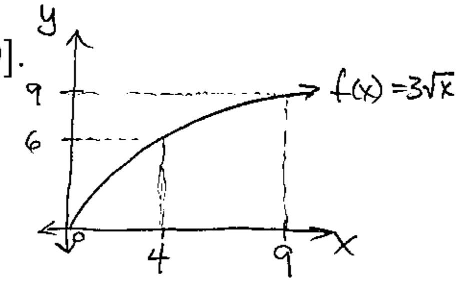
$$V_{\text{semi-circle}} = \frac{1}{2}\pi r^2 \Delta x$$

$$= \frac{1}{2}\pi\left[\frac{x-x^2}{2}\right]^2 \Delta x$$

$$= \frac{1}{2}\pi \cdot \frac{1}{4}(x-x^2)^2 \Delta x$$

$$= \frac{\pi}{8}(x^2 - 2x^3 + x^4)\Delta x$$

8. Find the average value of the function $f(x) = 3\sqrt{x}$ on the interval $[4,9]$.



$$f_{AVG} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f_{AVG} = \frac{1}{9-4}\int_4^9 3x^{\frac{1}{2}}\,dx$$

$$= \frac{1}{5}\cdot 3\cdot \frac{2}{3}x^{\frac{3}{2}}\Big|_4^9$$

$$= \frac{2}{5}\left[x^{3/2}\Big|_4^9\right]$$

$$= \frac{2}{5}\left[9^{3/2} - 4^{3/2}\right]$$

$$= \frac{2}{5}\left[27-8\right]$$

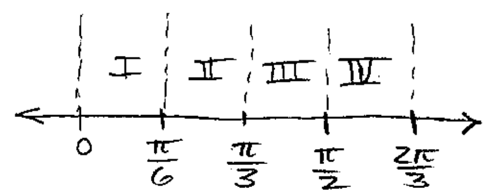
$$= \frac{2}{5}\left[19\right]$$

$$= \boxed{\frac{38}{5}}$$

9. Divide the specified interval into $n = 4$ subintervals of equal length and then compute $\sum_{k=1}^{4} f(x_k^*)\Delta x$

with x_k^* as the left endpoint of the subinterval. **Write your solution as a single fraction.**

$$f(x) = \cos x \quad \left[0, \frac{2\pi}{3}\right]$$



$$\Delta x = \frac{b-a}{n}$$

$$= \frac{\frac{2\pi}{3} - 0}{4}$$

$$= \frac{2\pi}{12}$$

$$= \frac{\pi}{6}$$

$$\sum_{k=1}^{4} f(x_k^*)\Delta x = f(x_1^*)\Delta x + f(x_2^*)\Delta x + f(x_3^*)\Delta x + f(x_4^*)\Delta x$$

$$= \frac{\pi}{6}\left[f(0) + f\left(\frac{\pi}{6}\right) + f\left(\frac{\pi}{3}\right) + f\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{\pi}{6}\left[\cos 0 + \cos\frac{\pi}{6} + \cos\frac{\pi}{3} + \cos\frac{\pi}{2}\right]$$

$$= \frac{\pi}{6}\left[1 + \frac{\sqrt{3}}{2} + \frac{1}{2} + 0\right]$$

$$= \frac{\pi}{6}\left[\frac{3}{2} + \frac{\sqrt{3}}{2}\right]$$

$$= \boxed{\frac{\pi(3+\sqrt{3})}{12}}$$