Calculus I

Practice Exam #04

12. Find the following.

a) Find $\left.\frac{dy}{dx}\right|_{x=\pi}$ if $y = x^{2\sin x}$.

b) Find $\left.\frac{dy}{dx}\right|_{x=\frac{4}{\pi}}$ if $y = x^2 \tan\left(\frac{1}{x}\right)$.

13. Find $\left.\frac{dy}{dx}\right|_{x=1}$ given $y = x^{e^{2x}}$.

14. Integrate.

a) $\displaystyle\int_{-\sqrt{e}/2}^{-1/2} \frac{5^{\ln(-2x)}}{2x}\, dx$

b) $\displaystyle\int_{\pi/12}^{\pi/4} (2\cos 2x)3^{2\sin 2x}\, dx$

12. Find the following.

a) Find $\left.\frac{dy}{dx}\right|_{x=\pi}$ if $y = x^{2\sin x}$.

$\ln y = 2\sin x \ln x$

$\frac{1}{y}\frac{dy}{dx} = 2\cos x \ln x + \frac{2\sin x}{x}$

$\frac{dy}{dx} = x^{2\sin x}\left[2\cos x \ln x + \frac{2\sin x}{x}\right]$

$\left.\frac{dy}{dx}\right|_{x=\pi} = \pi^{2\sin\pi}\left[2\cos\pi \ln\pi + \frac{2\sin\pi}{\pi}\right]$

$= \pi^{0}\left[-2\ln\pi + 0\right]$

$= -2\ln\pi$

$= \boxed{\ln\frac{1}{\pi^2}}$

b) Find $\left.\frac{dy}{dx}\right|_{x=\frac{4}{\pi}}$ if $y = x^2\tan\left(\frac{1}{x}\right)$.

$\frac{dy}{dx} = 2x\tan\left(\frac{1}{x}\right) + x^2\sec^2\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$

$\frac{dy}{dx} = 2x\tan\left(\frac{1}{x}\right) - \frac{x^2\sec^2\left(\frac{1}{x}\right)}{x^2}$

$\frac{dy}{dx} = 2x\tan\left(\frac{1}{x}\right) - \sec^2\left(\frac{1}{x}\right)$

$\left.\frac{dy}{dx}\right|_{x=\frac{4}{\pi}} = 2\left(\frac{4}{\pi}\right)\tan\frac{\pi}{4} - \sec^2\left(\frac{\pi}{4}\right)$

$= \frac{8}{\pi}(1) - \left(\sqrt{2}\right)^2$

$= \frac{8}{\pi} - 2$

$= \boxed{\frac{8-2\pi}{\pi}}$

13. Find $\left.\frac{dy}{dx}\right|_{x=1}$ given $y = x^{e^{2x}}$.

$\ln y = e^{2x}\ln x$

$\frac{1}{y}\frac{dy}{dx} = 2e^{2x}\ln x + e^{2x}\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = x^{e^{2x}}\left[2e^{2x}\ln x + \frac{e^{2x}}{x}\right]$

$\left.\frac{dy}{dx}\right|_{x=1} = 1^{e^2}\left[2(1^{e^2})\ln 1 + \frac{e^2}{1}\right]$

$= \boxed{e^2}$

14. Integrate.

a) $\int_{-\sqrt{e}/2}^{-1/2} \frac{5^{\ln(-2x)}}{2x}\,dx$

$u = \ln(-2x)$

$du = -\frac{1}{2x}(-2)\,dx$

$du = \frac{1}{x}\,dx$

$x = -\frac{1}{2} \rightarrow u = \ln(1) = 0$

$x = -\frac{\sqrt{e}}{2} \rightarrow u = \ln(\sqrt{e}) = \frac{1}{2}$

$\frac{1}{2}\int_{1/2}^{0} 5^u\,du$

$= \frac{1}{2}\left[\frac{5^u}{\ln 5}\Big|_{1/2}^{0}\right]$

$= \frac{1}{2\ln 5}\left[5^0 - 5^{1/2}\right]$

$= \boxed{\frac{1-\sqrt{5}}{2\ln 5} \text{ -OR- } \frac{-(\sqrt{5}-1)}{2\ln 5}}$

b) $\int_{\pi/12}^{\pi/4} (2\cos 2x)3^{2\sin 2x}\,dx$

$u = 2\sin 2x$

$du = 2\cos 2x(2)\,dx$

$du = 4\cos 2x\,dx$

$\frac{1}{2}du = 2\cos 2x\,dx$

$x = \frac{\pi}{4} \rightarrow u = 2\sin\left(\frac{\pi}{2}\right) = 2$

$x = \frac{\pi}{12} \rightarrow u = 2\sin\left(\frac{\pi}{6}\right) = 1$

$\frac{1}{2}\int_{1}^{2} 3^u\,du$

$= \frac{1}{2\ln 3}\left[3^u\Big|_{1}^{2}\right]$

$= \frac{3^2 - 3^1}{2\ln 3}$

$= \frac{9-3}{2\ln 3}$

$= \boxed{\frac{3}{\ln 3}}$