

Section
9.2

Exponential Function

What is an exponential function?

↙ Linear Function: $f(x) = 3x + 4$

↶ ↷ Quadratic Function: $f(x) = 2x^2 - 4x + 1$

↘ Exponential Function: $f(x) = 2^x$

* Notice the variable is in the exponent.

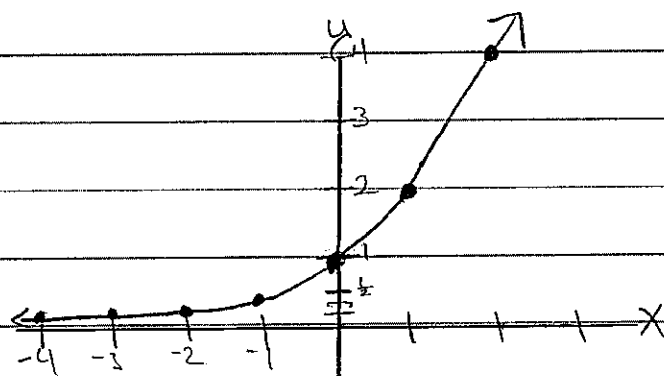
$f(x) = 2^x$ $f(0) = 2^0$
 $y = 1$

$0^0 = \text{undefined}$

$2^0 = 1$

$3^0 = 1$

$4^0 = 1$



$f(x) = 2^x$

Domain: \mathbb{R}

Range: $(0, \infty)$

This graph increases exponentially.

x	y
-4	$\frac{1}{16}$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16

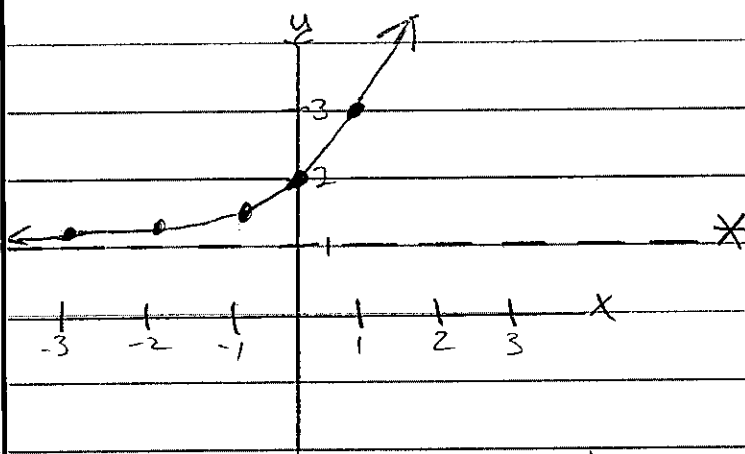
Example 1

Graph each Exponential Function

a) $f(x) = 2^x + 1$

$$f(x) = 2^{-1} + 1$$
$$= \frac{1}{2} + \frac{2}{2} = \frac{3}{2}$$

$$f(x) = 2^{-2} + 1$$
$$\frac{1}{4} + \frac{4}{4} = \frac{5}{4} \text{ or } 1\frac{1}{4}$$



x	y
-4	$1\frac{1}{6}$
-3	$1\frac{1}{8}$
-2	$1\frac{1}{4}$
-1	$\frac{3}{2} = 1\frac{1}{2}$
0	2
1	3
2	5
3	9
4	17

* The dashed line represents a horizontal asymptote.

Domain: $(-\infty, \infty)$

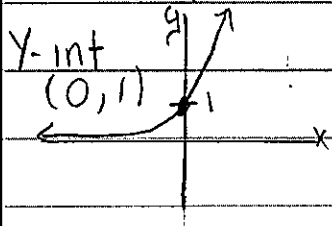
Range: $(1, \infty)$

Note: Given $f(x) = 2^x$

$$f(x) = 2^x + 2 \quad \text{up 2}$$

$$f(x) = 2^x + 1 \quad \text{up 1}$$

$f(x) = 2^x$ - this graph is not



"shifted" translated.

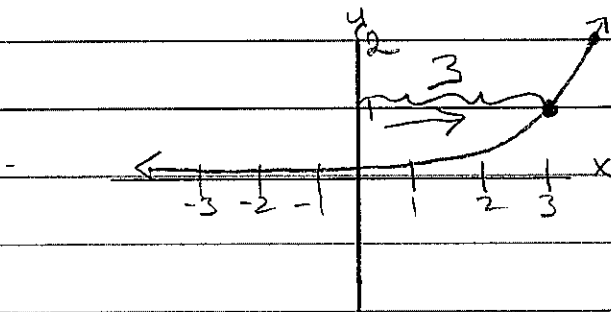
$$\left\{ \begin{array}{l} f(x) = 2^x - 1 \quad \text{down 1} \\ f(x) = 2^x - 2 \quad \text{down 2} \end{array} \right.$$

(Domain: \mathbb{R} Range: $(-2, +\infty)$)

These are called vertical shifts of $f(x) = 2^x$

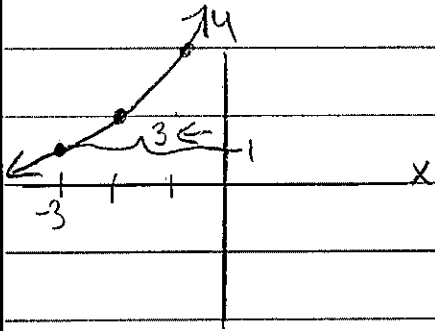
What about horizontal shifts?

$f(x) = 2^{x-3}$ (shifts graph 3 units to the right)



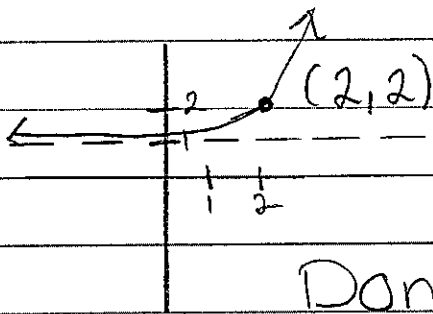
x	4
0	$\frac{1}{8}$
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	1
4	2
5	4
6	8

$$f(x) = 2^{(x+3)} \rightarrow \text{shifts left 3 units}$$



x	y
-7	$\frac{1}{16}$
-6	$\frac{1}{8}$
-5	$\frac{1}{4}$
-4	$\frac{1}{2}$
-3	1
-2	2
-1	4
0	8
1	16

$$f(x) = 3^{x-2} + 1 \text{ shifts right 2 + Up 1}$$



This is when the exponent is zero

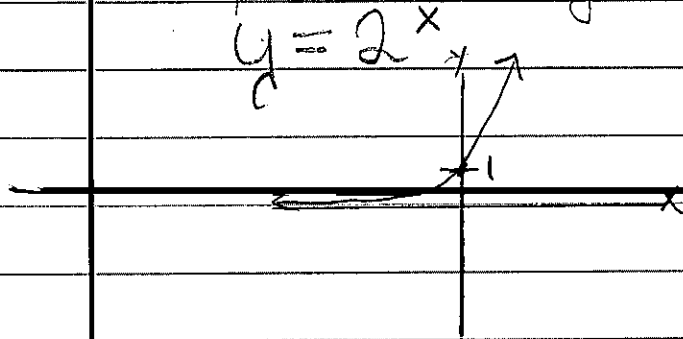
Domain: $(-\infty, \infty)$ "x"-values

Range: $(1, \infty)$ "y"-values

Now we will look at reflections about the x-axis and/or y-axis.

Exponential growth function

$$y = 2^x$$

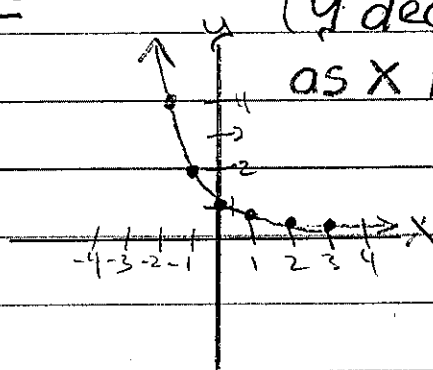


x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16

(y increases as x increases)

Exponential Decay Function

$$y = 2^{-x}$$



(y decreases as x increases)

x	y
3	$\frac{1}{8}$
2	$\frac{1}{4}$
1	$\frac{1}{2}$
0	1
-1	2
-2	4
-3	8

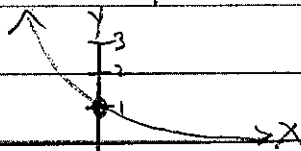
The negative quantity placed in front of the exponent causes a y-axis reflection.

Note: Negative exponents cause decay.

Graph $f(x) = \left(\frac{1}{4}\right)^x$

$$f(x) = \left(\frac{4}{1}\right)^{-x}$$

$$f(x) = 4^{-x}$$



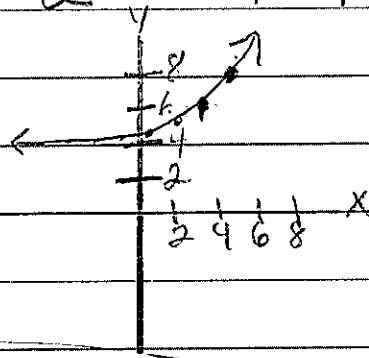
x	y
3	$\frac{1}{64}$
2	$\frac{1}{16}$
1	$\frac{1}{4}$
0	1

Domain: \mathbb{R}

Range: $(0, \infty)$

Example 2

Give the domain and range of the function: $2^{x-3} + 4$



x	y
0	$4\frac{1}{8}$
1	$4\frac{1}{4}$
2	$4\frac{1}{2}$
3	5
4	6
5	8
6	12

$$f(x) = 2^{x-3} + 4$$

$$f(x) = 2^0 + 4$$

$$f(x) = 1 + 4$$

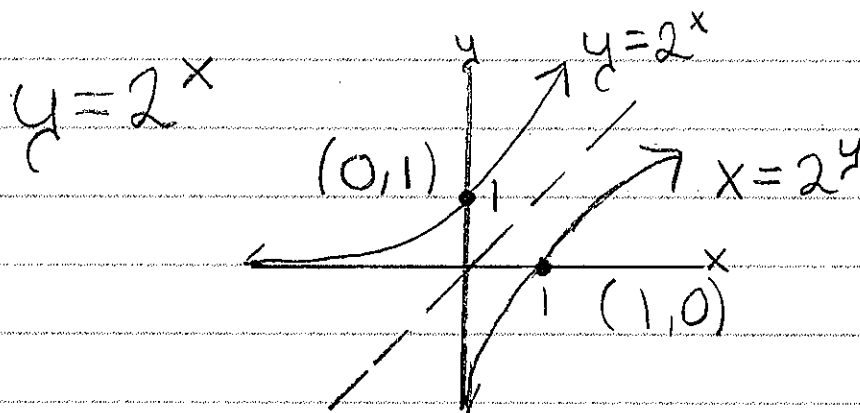
$$f(x) = 5$$

Domain: $(-\infty, \infty)$

Range: $(4, \infty)$

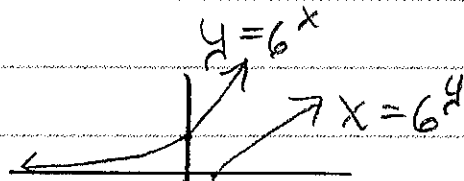
9.2 Recall

Switching "x with y" and "y with x" results in a reflection about the line $y=x$.



9.2

Homework #27
Graph $x=6^y$



Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

Homework #29
Graph $x=3^{-y}$

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

