

Math253

Practice Exam #02

1. Multiply the polynomials and simplify by combining like terms.

a) $(ab + c)(ab - c)$

b) $(r + 3)(r + 2)(r - 1)$

2. Factor the following trinomials.

b) $x^2 + 12xy + 27y^2$

b) $9a^2 + 18a + 8$

3. Factor completely.

a) $p^2q^2 - 16$

b) $(a+b)^2 - 25$

4. Factor the following.

a) $x^3 - 27$

b) $a^3 + \frac{1}{8}$

5. Solve each equation.

a) $(2t+5)(t-7)=0$

b) $9x^2-15x+4=0$

c) $4a^2-25=0$

6. Subtract and simplify.

$$\frac{4xy}{x^2 - y^2} - \frac{x - y}{x + y}$$

7. Solve the equation.

$$\frac{3}{x} + \frac{x}{x+2} = \frac{4}{x^2 + 2x}$$

8. Simplify each rational expression.

a)
$$\frac{\frac{a^2 - b^2}{ab}}{\frac{a - b}{b}}$$

b)
$$\frac{\frac{1}{x-2} + \frac{3}{x-1}}{\frac{2}{x-1} + \frac{5}{x+2}}$$

9. A Hudson River tug boat goes 10 mph in still water. It travels 24 mi upstream and 24 miles back downstream in a total time of 5 hours. What is the speed of the current?

10. The weight M of an object on mars varies directly as its weight E on earth. A person who weighs 95 pounds on earth weighs 38 pounds on mars. How much would a 100 pound person weigh on Mars?

Math253 – Exam #02 – Fall 2015 – Solutions

1. Multiply the polynomials and simplify by combining like terms.

a) $(ab+c)(ab-c)$

$$ab(ab+c) - c(ab+c)$$

$$a^2b^2 + abc - abc - c^2$$

$$\boxed{a^2b^2 - c^2}$$

b) $(r+3)(r+2)(r-1)$

$$(r^2+3r+2r+6)(r-1)$$

$$(r^2+5r+6)(r-1)$$

$$r^2(r-1) + 5r(r-1) + 6(r-1)$$

$$r^3 - r^2 + 5r^2 - 5r + 6r - 6$$

$$\boxed{r^3 + 4r^2 + r - 6}$$

2. Factor the following trinomials.

b) $x^2 + 12xy + 27y^2$

$$\begin{array}{r} 27 \\ \wedge \\ 27 \ 1 \\ 9 \ 3 \end{array}$$

$$\underline{x^2 + 9xy + 3xy + 27y^2}$$

$$\underline{x(x+9y) + 3y(x+9y)}$$

$$\boxed{(x+9y)(x+3y)}$$

b) $9a^2 + 18a + 8$

$$\begin{array}{r} 72 \\ \wedge \\ 72 \ 1 \\ 36 \ 2 \\ 18 \ 4 \\ 9 \ 8 \\ 3 \ 24 \\ 6 \ 12 \end{array}$$

$$\underline{9a^2 + 12a + 6a + 8}$$

$$\underline{3a(3a+4) + 2(3a+4)}$$

$$\boxed{(3a+4)(3a+2)}$$

3. Factor completely.

a) $p^2q^2 - 16$

$$(pq)^2 - (4)^2$$

$$(pq+4)(pq-4)$$

b) $(a+b)^2 - 25$

$$(a+b)^2 - (5)^2$$

$$(a+b+5)(a+b-5)$$

4. Factor the following.

a) $x^3 - 27$

$$x^3 - (3)^3$$

$$(x-3)(x^2+3x+9)$$

b) $a^3 + \frac{1}{8}$

$$a^3 + \left(\frac{1}{2}\right)^3$$

$$\left(a + \frac{1}{2}\right)\left(a^2 - \frac{a}{2} + \frac{1}{4}\right)$$

5. Solve each equation.

a) $(2t+5)(t-7)=0$

$$\begin{array}{r} 2t+5=0 \\ -5 \quad -5 \\ \hline 2t = -5 \\ \frac{2t}{2} = \frac{-5}{2} \\ \boxed{t = -\frac{5}{2}} \end{array} \quad \begin{array}{r} t-7=0 \\ +7 \quad +7 \\ \hline \boxed{t=7} \end{array}$$

b) $9x^2-15x+4=0$

36
36 1
18 2
9 4
3 12
-3 -12

$$9x^2 - 3x - 12x + 4 = 0$$

$$3x(3x-1) - 4(3x-1) = 0$$

$$(3x-1)(3x-4) = 0$$

$$\begin{array}{r} 3x-1=0 \\ +1 \quad +1 \\ \hline 3x = 1 \\ \frac{3x}{3} = \frac{1}{3} \\ \boxed{x = \frac{1}{3}} \end{array}$$

$$\begin{array}{r} 3x-4=0 \\ +4 \quad +4 \\ \hline 3x = 4 \\ \frac{3x}{3} = \frac{4}{3} \\ \boxed{x = \frac{4}{3}} \end{array}$$

c) $4a^2-25=0$

$$(2a)^2 - (5)^2 = 0$$

$$(2a+5)(2a-5) = 0$$

$$\begin{array}{r} 2a+5=0 \\ -5 \quad -5 \\ \hline 2a = -5 \\ \frac{2a}{2} = \frac{-5}{2} \\ \boxed{a = -\frac{5}{2}} \end{array} \quad \begin{array}{r} 2a-5=0 \\ +5 \quad +5 \\ \hline 2a = 5 \\ \frac{2a}{2} = \frac{5}{2} \\ \boxed{a = \frac{5}{2}} \end{array}$$

6. Subtract and simplify.

$$\frac{4xy}{x^2-y^2} - \frac{x-y}{x+y}$$

$$\frac{4xy}{(x+y)(x-y)} - \frac{x-y}{x+y}$$

LCD = (x+y)(x-y)

$$\frac{4xy}{(x+y)(x-y)} - \frac{x-y}{x+y} \left(\frac{x-y}{x-y} \right)$$

$$\frac{4xy}{(x+y)(x-y)} - \frac{(x-y)^2}{(x+y)(x-y)}$$

$$\frac{4xy - (x-y)^2}{(x+y)(x-y)}$$

$$\frac{4xy - [x^2 - 2xy + y^2]}{(x+y)(x-y)}$$

$$\frac{(4xy - x^2 + 2xy - y^2)}{(x+y)(x-y)}$$

$$\frac{-x^2 + 6xy - y^2}{(x+y)(x-y)}$$

-OR-

$$-\frac{(x^2 - 6xy + y^2)}{(x+y)(x-y)}$$

7. Solve the equation.

$$\frac{3}{x} + \frac{x}{x+2} = \frac{4}{x^2+2x}$$

$$x \neq 0$$

$$x \neq -2$$

$$\frac{3}{x} + \frac{x}{x+2} = \frac{4}{x(x+2)}$$

LCD = x(x+2)

$$x(x+2) \left[\frac{3}{x} \right] + x(x+2) \left[\frac{x}{x+2} \right] = x(x+2) \left[\frac{4}{x(x+2)} \right]$$

$$3(x+2) + x(x) = 4$$

$$3x + 6 + x^2 = 4$$

$$x^2 + 3x + 6 = 4$$

-4 -4

$$x^2 + 3x + 2 = 0$$

$$\bigwedge_{z=1}^z (x+2)(x+1) = 0$$

$$\frac{x+2=0}{-2 \quad -2}$$

$$\boxed{\cancel{x=-2}}$$

$$\underline{\underline{x \neq -2}}$$

$$\frac{x+1=0}{-x-1}$$

$$\boxed{x=-1}$$

8. Simplify each rational expression.

a) $\frac{\frac{a^2-b^2}{ab}}{\frac{a-b}{b}}$ LCD = ab

$$\left[\frac{\frac{a^2-b^2}{ab}}{\frac{a-b}{b}} \right] \frac{ab}{ab}$$

$$\frac{a^2-b^2}{a(a-b)}$$

$$\frac{(a+b)(a-b)}{a(a-b)}$$

$$\boxed{\frac{a+b}{a}}$$

$$\text{LCD} = (x-2)(x-1)(x+2)$$

b) $\left[\frac{\frac{1}{x-2} + \frac{3}{x-1}}{\frac{2}{x-1} + \frac{5}{x+2}} \right] \frac{(x-2)(x-1)(x+2)}{(x-2)(x-1)(x+2)}$

$$\frac{[(x-1)(x+2) + 3(x-2)(x+2)]}{[2(x-2)(x+2) + 5(x-2)(x-1)]}$$

GCF \rightarrow $\frac{(x+2)[(x-1) + 3(x-2)]}{(x-2)[2(x+2) + 5(x-1)]}$

GCF \rightarrow

$$\frac{(x+2)[x-1+3x-6]}{(x-2)[2x+4+5x-5]}$$

$$\boxed{\frac{(x+2)(4x-7)}{(x-2)(7x-1)}}$$

9. A Hudson River tug boat goes 10 mph in still water. It travels 24 mi upstream and 24 miles back downstream in a total time of 5 hours. What is the speed of the current?

$$R_{\text{Boat}} = 10 \text{ mph}$$

$$R_{\text{CURRENT}} = x$$

	R	x	t	= Dist
UPSTREAM	10-x		$\frac{24}{10-x}$	24
DOWNSTREAM	10+x		$\frac{24}{10+x}$	24

$$t_{\text{up}} + t_{\text{down}} = 5$$

$$\frac{24}{10-x} + \frac{24}{10+x} = 5$$

$$\text{LCD} = (10-x)(10+x)$$

$$(10-x)(10+x) \left[\frac{24}{10-x} \right] + (10-x)(10+x) \left[\frac{24}{10+x} \right] = 5(10-x)(10+x)$$

$$24(10+x) + 24(10-x) = 5(10-x)(10+x)$$

$$240 + 24x + 240 - 24x = 5(100 - x^2)$$

$$480 = 500 - 5x^2$$

$$+ 5x^2 \quad + 5x^2$$

$$5x^2 + 480 = 500$$

$$-480 \quad -480$$

$$5x^2 = \frac{20}{5}$$

$$R_{\text{CURRENT}} = 2 \text{ mph}$$

$$\sqrt{x^2} = \pm \sqrt{4}$$

$$x = \pm 2$$

10. The weight M of an object on mars varies directly as its weight E on earth. A person who weighs 95 pounds on earth weighs 38 pounds on mars. How much would a 100 pound person weigh on Mars?

$$M = kE$$

$$M = 38 \text{ when } E = 95$$

$$\frac{38}{95} = \frac{k \cdot 95}{95}$$

$$\frac{38/19}{95/19} = k$$

$$\frac{2}{5} = k$$

$$M = \frac{2}{5} E$$

$$\text{Find } M \text{ when } E = 100$$

$$M = \frac{2}{5} (100)$$

$$M = 40 \text{ pounds}$$