

**§3-3****ABSOLUTE VALUE EQUATIONS****Theorem**

If  $|u| = c$  where  $u$  is a variable expression and  $c$  is a non-negative real number, then  $u = c$  or  $u = -c$ .

**Example 1** Solve the equation for  $x$ :  $|x| = 6$ .

**Solution**  $x = 6$  or  $x = -6$

**Example 2** Solve the equation for  $x$ :  $|x + 3| = 5$ .

**Solution**  $x + 3 = 5$  or  $x + 3 = -5$   
 $x = 2$   $x = -8$

**Example 3** Solve the equation for  $y$ :  $|2y - 4| = 7$ .

**Solution**  $2y - 4 = 7$  or  $2y - 4 = -7$   
 $2y = 11$   $2y = -3$   
 $y = \frac{11}{2}$   $y = -\frac{3}{2}$

**Example 4** Solve the equation for  $x$ :  $|3x + 2| - 13 = 7$ .

**Solution**  $|3x + 2| - 13 = 7$   
 $|3x + 2| = 20$   
 $3x + 2 = 20$  or  $3x + 2 = -20$   
 $3x = 18$   $3x = -22$   
 $x = 6$   $x = -\frac{22}{3}$

**Example 5** Solve the equation for  $t$ :  $|t^2 + 2| = 7$ .

**Solution**  $t^2 + 2 = 7$  or  $t^2 + 2 = -7$   
 $t^2 = 5$   $t^2 = -9$   
 $\sqrt{t^2} = \pm\sqrt{5}$   $\sqrt{t^2} = \pm\sqrt{-9}$   
 $t = \pm\sqrt{5}$   $t = \pm 3i$

So  $t = \pm\sqrt{5}$  are the only real solutions.

Solve each equation. If there is no solution then write *no solution*.

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| 1. $ x  = 5$                               | 2. $ x  = 0$                                      | 3. $ 3v  = \frac{5}{2}$                  |
| 4. $ x  + 3 = 5$                           | 5. $ p  - \frac{1}{2} = \frac{1}{4}$              | 6. $ x  = -2$                            |
| 7. $3 t  = 12$                             | 8. $ 3t  = 12$                                    | 9. $\left \frac{y}{3}\right  = 5$        |
| 10. $\frac{1}{3} w  = 4$                   | 11. $ x - 4  = 2$                                 | 12. $ 5x + 1  = 10$                      |
| 13. $ 3a - a  = 3$                         | 14. $\frac{1}{2} 2x + 1  = 15$                    | 15. $ -2x + 1  = 8$                      |
| 16. $ 7z + 1  = 2$                         | 17. $4 3a - 1  - 4 = -3$                          | 18. $\left \frac{2}{3}x + 4\right  = 16$ |
| 19. $ 1 + 3q  = \frac{1}{5}$               | 20. $ 3x + 1  + 6 = 10$                           | 21. $ 3x - 1  = 4$                       |
| 22. $ 3x + 2  - 4 = 4$                     | 23. $5 4x - 3  + 7 = 52$                          | 24. $\left \frac{1}{2}t\right  = 4$      |
| 25. $ 3n + 4  = \frac{1}{2}$               | 26. $ 4x^2 - 9  = 0$                              | 27. $ y - 5  = 3$                        |
| 28. $ -6y - 3  = 4$                        | 29. $\left \frac{3x - 2}{5}\right  = 1$           | 30. $ x^2 - 2  = 5x$                     |
| 31. $ 3t - 1  =  t - 4 $                   | 32. $\left \frac{4}{5}r + 9\right  = \frac{1}{3}$ | 33. $3 h - 14  - 7 = 2$                  |
| 34. $-2\left \frac{4s - 3}{5}\right  = -1$ | 37. $ y - 3  = 5$                                 | 36. $2 x + 2  = 4$                       |
| 37. $ 2p  =  p - 4 $                       | 38. $\left 4 - \frac{b}{2}\right  = 3$            | 39. $ 2x + 3  = 11$                      |
| 40. $2 -  t - 5  = 4$                      | 41. $8 -  1 - 3s  = -1$                           | 42. $\frac{1}{3} 3a - 7  + 8 = 10$       |

Solve for  $x$ .

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| 43. $ ax + b  = y$ | 44. $a x  + b = y$ | 45. $ 3x + y  = 4$ |
|--------------------|--------------------|--------------------|

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| 1. $\{-5, 5\}$                                     | 2. $\{0\}$  | 3. $\left\{-\frac{5}{6}, \frac{5}{6}\right\}$                          |
| 4. $\{-2, 2\}$                                     | 5. $\left\{-\frac{3}{4}, \frac{3}{4}\right\}$     | 6. no solution   |
| 7. $\{-4, 4\}$                                     | 8. $\{-4, 4\}$                                    | 9. $\{-15, 15\}$   |
| 10. $\{-12, 12\}$                                  | 11. $\{2, 6\}$                                    | 12. $\left\{-\frac{11}{5}, \frac{9}{5}\right\}$                        |
| 13. $\left\{-\frac{3}{2}, \frac{3}{2}\right\}$     | 14. $\left\{-\frac{31}{2}, \frac{29}{2}\right\}$  | 15. $\left\{-\frac{7}{2}, \frac{9}{2}\right\}$                         |
| 16. $\left\{-\frac{3}{7}, \frac{1}{7}\right\}$     | 17. $\left\{\frac{1}{4}, \frac{5}{12}\right\}$    | 18. $\{-30, 18\}$  |
| 19. $\left\{-\frac{4}{15}, -\frac{2}{5}\right\}$   | 20. $\left\{-\frac{5}{3}, 1\right\}$              | 21. $\left\{-1, \frac{5}{3}\right\}$                                   |
| 22. $\left\{-\frac{10}{3}, 2\right\}$              | 23. $\left\{-\frac{3}{2}, 3\right\}$              | 24. $\{-8, 8\}$  |
| 25. $\left\{-\frac{7}{6}, -\frac{3}{2}\right\}$    | 26. $\left\{\pm\frac{3}{2}\right\}$               | 27. $\{2, 8\}$   |
| 28. $\left\{-\frac{7}{6}, \frac{1}{6}\right\}$     | 29. $\left\{-1, \frac{7}{3}\right\}$              | 30. $\left\{\frac{5\pm\sqrt{33}}{2}, \frac{-5\pm\sqrt{33}}{2}\right\}$ |
| 31. $\left\{-\frac{3}{2}, \frac{5}{4}\right\}$     | 32. $\left\{-\frac{65}{6}, -\frac{35}{3}\right\}$ | 33. $\{11, 17\}$   |
| 34. $\left\{\frac{1}{8}, \frac{11}{8}\right\}$     | 37. $\{-2, 8\}$                                   | 36. $\{-4, 0\}$  |
| 37. $\left\{-4, \frac{4}{3}\right\}$               | 38. $\{2, 14\}$                                   | 39. $\{-7, 4\}$  |
| 40. no solution                                    | 41. $\left\{-\frac{8}{3}, \frac{10}{3}\right\}$   | 42. $\left\{\frac{1}{3}, \frac{13}{3}\right\}$                         |
| 43. $\left\{-\frac{y+b}{a}, \frac{y-b}{a}\right\}$ | 44. $\left\{\pm\frac{y-b}{a}\right\}$             | 45. $\left\{-\frac{4+y}{3}, \frac{4-y}{3}\right\}$                     |