

§3-4**RATIONAL EQUATIONS****Definition**

Rational equations are equations involving rational expressions.

The technique for solving rational equations is called clearing the fractions.

Procedure**Clearing the Fractions**

1. Factor all polynomial expressions.
2. Multiply through by the common denominator of all the terms.
3. Reduce wherever possible
4. Solve the resulting equation
5. Check your answers using the original equation

Example 1

Solve $\frac{2}{x+1} - \frac{1}{x^2+x} = \frac{3}{x}$

Solution

$$\frac{2}{x+1} - \frac{1}{x^2+x} = \frac{3}{x}$$

Step 1

$$\frac{2}{x+1} - \frac{1}{x(x+1)} = \frac{3}{x}$$

Step 2 $x(x+1)\left(\frac{2}{x+1}\right) - x(x+1)\left(\frac{1}{x(x+1)}\right) = x(x+1)\left(\frac{3}{x}\right)$

Step 3

$$2x - 1 = 3(x + 1)$$

Step 4

$$\begin{aligned} 2x - 1 &= 3x + 3 \\ 2x - 3x &= 3 + 1 \\ -x &= 4 \\ x &= -4 \end{aligned}$$

Step 5

$$\begin{aligned} \frac{2}{x+1} - \frac{1}{x^2+x} &? \frac{3}{x} \\ \frac{2}{(-4)+1} - \frac{1}{(-4)^2+(-4)} &? \frac{3}{(-4)} \\ -\frac{2}{3} - \frac{1}{12} &? -\frac{3}{4} \\ -\frac{8}{12} - \frac{1}{12} &? -\frac{3}{4} \\ -\frac{9}{12} &? -\frac{3}{4} \\ -\frac{3}{4} &= -\frac{3}{4} \end{aligned}$$

So -4 is the solution of $\frac{2}{x+1} - \frac{1}{x^2+x} = \frac{3}{x}$.

Example 2 Solve $\frac{x}{5x+5} = \frac{1}{x+2} + \frac{1}{x^2+3x+2}$.

Solution

$$\frac{x}{5x+5} = \frac{1}{x+2} + \frac{1}{x^2+3x+2}$$

$$\frac{x}{5(x+1)} = \frac{1}{x+2} + \frac{1}{(x+1)(x+2)}$$

$$5(x+1)(x+2)\left(\frac{x}{5(x+1)}\right) = 5(x+1)(x+2)\left(\frac{1}{x+2}\right) + 5(x+1)(x+2)\left(\frac{1}{(x+1)(x+2)}\right)$$

$$x(x+2) = 5(x+1) + 5$$

$$x^2 + 2x = 5x + 5 + 5$$

$$x^2 + 2x = 5x + 10$$

$$x^2 - 3x - 10 = 0$$

$$(x+2)(x-5) = 0$$

$$x+2=0 \text{ or } x-5=0$$

$$x=-2 \quad x=5$$

$$\frac{x}{5x+5} \stackrel{?}{=} \frac{1}{x+2} + \frac{1}{x^2+3x+2}$$

$$\frac{(-2)}{5(-2)+5} \stackrel{?}{=} \frac{1}{(-2)+2} + \frac{1}{(-2)^2+3(-2)+2}$$

$$\frac{2}{5} \stackrel{?}{=} \frac{1}{0} + \frac{1}{0}$$

$$\frac{2}{5} \neq \text{undefined}$$

So -2 is not a solution of $\frac{x}{5x+5} = \frac{1}{x+2} + \frac{1}{x^2+3x+2}$.

$$\frac{x}{5x+5} \stackrel{?}{=} \frac{1}{x+2} + \frac{1}{x^2+3x+2}$$

$$\frac{(5)}{5(5)+5} \stackrel{?}{=} \frac{1}{(5)+2} + \frac{1}{(5)^2+3(5)+2}$$

$$\frac{5}{30} \stackrel{?}{=} \frac{1}{7} + \frac{1}{42}$$

$$\frac{1}{6} \stackrel{?}{=} \frac{6}{42} + \frac{1}{42}$$

$$\frac{1}{6} \stackrel{?}{=} \frac{7}{42}$$

$$\frac{1}{6} = \frac{1}{6}$$

So 5 is the only solution of $\frac{x}{5x+5} = \frac{1}{x+2} + \frac{1}{x^2+3x+2}$.

Solve each equation for x .

1. $a + b = \frac{c+a}{x}$

2. $\frac{1}{a} - \frac{2}{x} = \frac{3}{b}$

3. $\frac{a}{x} + 1 = \frac{2}{x}$

4. $\frac{1}{a} + \frac{1}{b} = \frac{c}{x}$

Solve each rational equation. If there is no solution then write *no solution*.

5. $\frac{x+1}{5} = \frac{x+3}{3}$

6. $a + \frac{25}{a} = 10$

7. $\frac{4}{b-4} - \frac{3}{b-3} = 1$

8. $\frac{1}{t^2} - 16 = 0$

9. $\frac{1}{x-3} = \frac{8}{x^2-9}$

10. $\frac{5}{x-2} - \frac{2}{x+2} = \frac{3}{x^2-4}$

11. $\frac{3}{y} = 2 + \frac{1}{y}$

12. $\frac{5}{x^2-7x+12} = \frac{2}{x-3} + \frac{5}{x-4}$

13. $\frac{4}{y-4} - \frac{3}{y-3} = 1$

14. $\frac{3}{2} - \frac{z}{5} = \frac{1}{10} + \frac{3z}{20}$

15. $1 - \frac{3}{b} = \frac{10}{b^2}$

16. $\frac{3}{x^2-16} + \frac{1}{2x+8} = 0$

17. $\frac{x+2}{x^2-4} = \frac{3}{x-6}$

18. $\frac{x}{x-4} + \frac{6}{x-3} = \frac{16}{(x-4)(x-3)}$

19. $\frac{8}{a^2} + 1 = \frac{9}{a}$

20. $\frac{2}{x+2} + \frac{1}{x-2} = \frac{3}{x}$

21. $x - \frac{12}{x} = 1$

22. $5 - \frac{2}{2x-2} = \frac{3}{x^2-4}$

23. $\frac{3}{x+2} = \frac{4}{x-1}$

24. $\frac{x}{x^2-1} + \frac{2}{x+1} = \frac{1}{2x-2}$

25. $\frac{2}{p} = 3 + \frac{1}{p}$

26. $\frac{2}{4t^2-9} + \frac{1}{2t-3} = \frac{3}{2t+3}$

27. $\frac{1}{x-2} + \frac{2}{x(x-1)} + \frac{2}{x(x-1)(x-2)} = 0$

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|----------------------------------|-----------------------|-----------------|-----------------------|
| 1. $\frac{a+c}{a+b}$ | 2. $\frac{2ab}{b-3a}$ | 3. $2 - a$ | 4. $\frac{2ab}{b-3a}$ |
| 5. $\{-6\}$ | 6. 5 | 7. $\{2, 6\}$ | 8. $\pm\frac{1}{4}$ |
| 9. $\{5\}$ | 10. $-\frac{11}{3}$ | 11. $\{1\}$ | 12. no solution |
| 13. $\{2, 6\}$ | 14. 4 | 15. $\{-2, 5\}$ | 16. -2 |
| 17. $\{0\}$ | 18. $\{-8, 5\}$ | 19. $\{8, 1\}$ | 20. 6 |
| 21. $\{-3, 4\}$ | 22. SKIP | 23. $\{-11\}$ | 24. no solution |
| 25. $\left\{\frac{1}{3}\right\}$ | 26. $\frac{7}{2}$ | 27. $\{-2\}$ | |