

## §6-4

## ANGLES IN A PLANE

### Definition

A pair of **supplementary angles** have measures which add to  $180^\circ$ .

### Definition

A pair of **complementary angles** have measures which add to  $90^\circ$ .

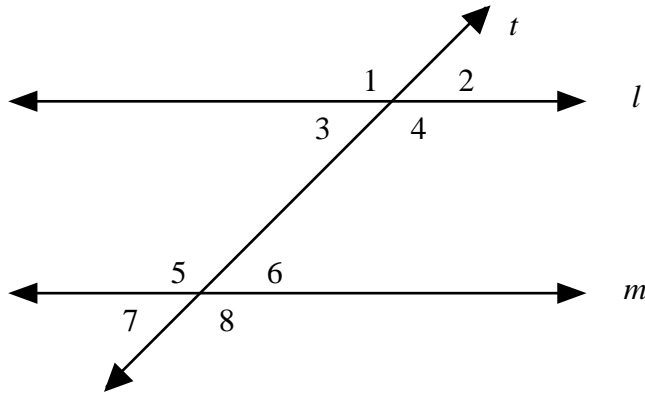
### Definition

**Parallel lines** are lines that lie in a same plane (**coplanar**) but do not intersect.

### Definition

A **transversal** is a line that intersects two coplanar lines.

The figure below represents a transversal intersecting two parallel lines to form eight angles. The region between the lines  $l$  and  $m$  is called the **interior region**. The region not between lines  $l$  and  $m$  is called the **exterior region**. Angles that are on opposite sides of the transversal are referred to as **alternate angles**.



### Properties

<u>Angle Pair</u>	<u>Measures</u>	<u>Examples</u>
Alternate Interior	equal	$\angle 4$ and $\angle 5$ ; $\angle 3$ and $\angle 6$
Same Side Interior	supplementary	$\angle 3$ and $\angle 5$ ; $\angle 4$ and $\angle 6$
Alternate Exterior	equal	$\angle 1$ and $\angle 8$ ; $\angle 2$ and $\angle 7$
Same Side Exterior	supplementary	$\angle 1$ and $\angle 7$ ; $\angle 2$ and $\angle 8$
Corresponding	equal	$\angle 1$ and $\angle 5$ ; $\angle 2$ and $\angle 6$ $\angle 3$ and $\angle 7$ ; $\angle 4$ and $\angle 8$
Vertical	equal	$\angle 1$ and $\angle 4$ ; $\angle 2$ and $\angle 3$ $\angle 5$ and $\angle 8$ ; $\angle 6$ and $\angle 7$

**Example 1** If  $m\angle 1 = 115^\circ$  in the previous diagram, then find the measures of the other angles.

**Solution**

$\angle 1$  and  $\angle 2$  are supplementary and so  $m\angle 2 = 180^\circ - 115^\circ = 65^\circ$ .

$\angle 1$  and  $\angle 3$  are supplementary and so  $m\angle 3 = 180^\circ - 115^\circ = 65^\circ$ .

$\angle 1$  and  $\angle 4$  are vertical angles and so  $m\angle 4 = 115^\circ$ .

$\angle 1$  and  $\angle 5$  are corresponding angles and so  $m\angle 5 = 115^\circ$ .

$\angle 5$  and  $\angle 6$  are supplementary and so  $m\angle 6 = 180^\circ - 115^\circ = 65^\circ$ .

$\angle 1$  and  $\angle 7$  are same side exterior angles and so  $m\angle 7 = 180^\circ - 115^\circ = 65^\circ$ .

$\angle 1$  and  $\angle 8$  are alternate exterior angles and so  $m\angle 8 = 115^\circ$ .

Thus we see that  $m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8 = 115^\circ$  and

$$m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7 = 65^\circ$$

**Theorem**

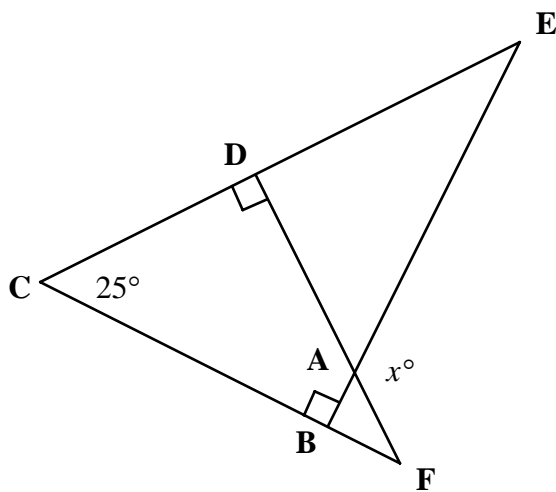
**The Angle-Sum Theorem for Polygons**

The sum,  $S$ , of the measures of the interior angles of a convex polygon with  $n$  sides is:

$$S = (n - 2)180^\circ$$

**Example 2**

In the figure below, find  $x$ .



**Solution**

By the angle-sum theorem, the polygon sum of the interior angles of polygon **ABCD** is  $(4 - 2)180^\circ = 2 \cdot 180^\circ = 360^\circ$ .

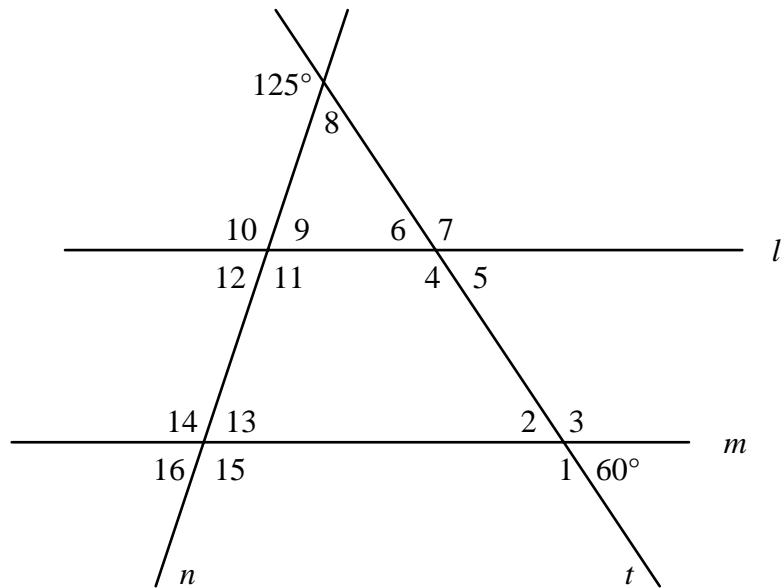
Since  $m\angle C = 25^\circ$ ,  $m\angle CDA = 90^\circ$  and  $m\angle CBA = 90^\circ$  there are  $205^\circ$  accounted for.

Thus  $m\angle DAB = 360^\circ - 205^\circ = 155^\circ$ .

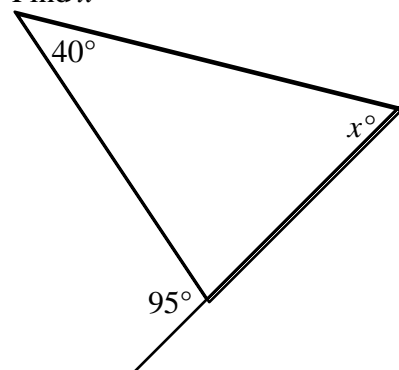
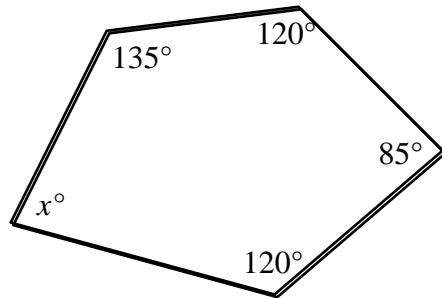
Since  $\angle DAB$  and  $\angle EAF$  are vertical angles, they must have the same measure.

Therefore  $x = 155$ .

In the figure below the lines  $l$  and  $m$  are parallel. Find the measure of each angle.



1.  $m\angle 1$
2.  $m\angle 2$
3.  $m\angle 3$
4.  $m\angle 4$
5.  $m\angle 5$
6.  $m\angle 6$
7.  $m\angle 7$
8.  $m\angle 8$
9.  $m\angle 9$
10.  $m\angle 10$
11.  $m\angle 11$
12.  $m\angle 12$
13.  $m\angle 13$
14.  $m\angle 14$
15.  $m\angle 15$
16.  $m\angle 16$
17. Find  $x$
18. Find  $x$



19. What is the sum of the interior angles of a convex pentagon (5-sided polygon)?
20. What is the sum of the interior angles of a convex octagon (8-sided polygon)?
21. What is the measure of each interior angle of a convex, regular, 25 sided polygon?
22. What is the measure of each interior angle of a convex, regular, 30 sided polygon?

- |                  |                  |                 |                  |
|------------------|------------------|-----------------|------------------|
| <b>1.</b> 120°   | <b>2.</b> 60°    | <b>3.</b> 120°  | <b>4.</b> 120°   |
| <b>5.</b> 60°    | <b>6.</b> 60°    | <b>7.</b> 120°  | <b>8.</b> 55°    |
| <b>9.</b> 65°    | <b>10.</b> 115°  | <b>11.</b> 115° | <b>12.</b> 65°   |
| <b>13.</b> 65°   | <b>14.</b> 115°  | <b>15.</b> 115° | <b>16.</b> 65°   |
| <b>17.</b> 80°   | <b>18.</b> 55°   | <b>19.</b> 540° | <b>20.</b> 1080° |
| <b>21.</b> 4140° | <b>22.</b> 5040° |                 |                  |