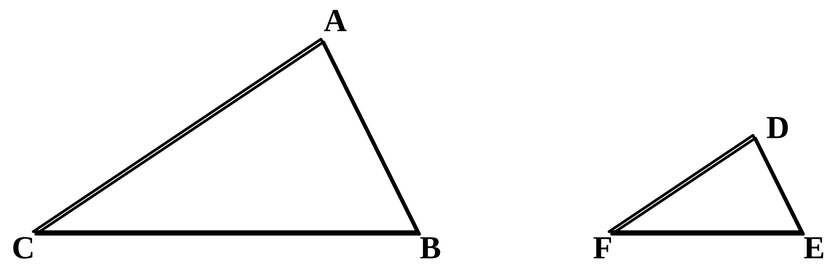
§6-5 SPECIAL TRIANGLES

Definition

Two triangles are **similar** if corresponding angles are equal in measure. For similar triangles corresponding sides are in proportion.



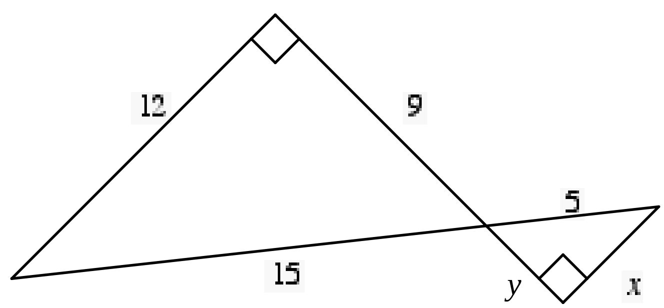
If triangle **ABC** and triangle **DEF** are similar (Δ**ABC** ~ Δ**DEF**) then

$m\angle\mathbf{A} = m\angle\mathbf{D}$, $m\angle\mathbf{B} = m\angle\mathbf{E}$, $m\angle\mathbf{C} = m\angle\mathbf{F}$ and $\frac{\mathbf{AB}}{\mathbf{DE}} = \frac{\mathbf{BC}}{\mathbf{EF}} = \frac{\mathbf{CA}}{\mathbf{FD}}$.

Theorem

If a pair of triangles has two pairs of congruent corresponding angles then they are similar.

Example 1 In the figure below, find *x* and *y*.



Solution Where the triangles meet they form vertical angles. Since the vertical angles are equal in measure and each triangle also has one right angle, the two triangles are similar. Since the ratios of corresponding sides are equal we have the following proportion.

$$\frac{x}{12} = \frac{5}{15}$$

Solving for *x* produces…

$$\begin{aligned} 15x &= 12 \cdot 5 \\ 15x &= 60 \\ x &= 4 \end{aligned}$$

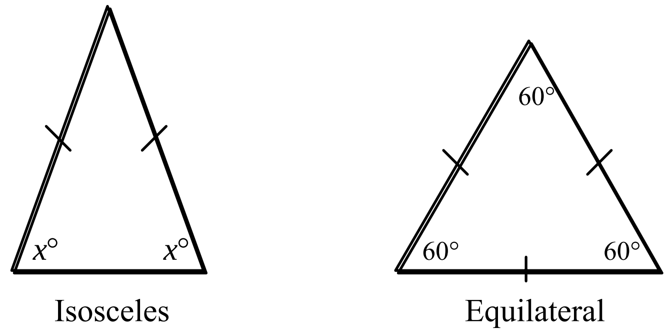
Similarly since the sides of the small triangle are one-third the length of their corresponding sides in the larger triangle, *y* = 3.

Definition

A triangle with two equal sides is called an **isosceles triangle**. The angles opposite the two equal sides are equal in measure.

Definition

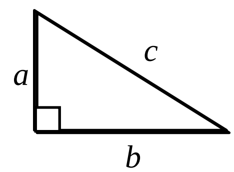
A triangle with three equal sides is called an **equilateral triangle**. All three angles of an equilateral triangle are $60°$.



Theorem

Pythagorean Theorem

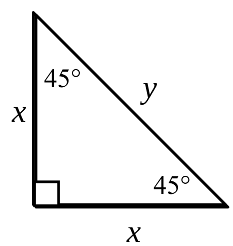
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



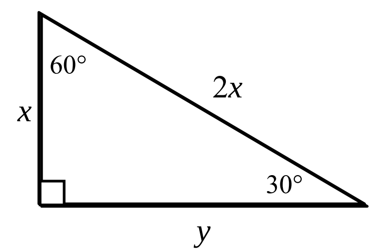
$$c^2 = a^2 + b^2$$

Example 2 Find y for each of the triangles below.

a.



b.



Solution

a.

$$\begin{aligned} y^2 &= x^2 + x^2 \\ y^2 &= 2x^2 \\ \sqrt{y^2} &= \sqrt{2x^2} \\ y &= x\sqrt{2} \end{aligned}$$

b.

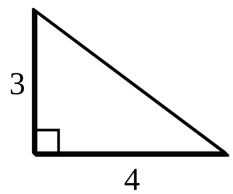
$$\begin{aligned} x^2 + y^2 &= (2x)^2 \\ x^2 + y^2 &= 4x^2 \\ y^2 &= 3x^2 \\ \sqrt{y^2} &= \sqrt{3x^2} \\ y &= x\sqrt{3} \end{aligned}$$

These triangles are examples of two common types named after their angle measures: a 45-45-90 triangle and a 30-60-90 triangle. A 45-45-90 triangle is an isosceles right triangle and has a hypotenuse which is $\sqrt{2}$ times as long as either leg. In a 30-60-90 triangle the hypotenuse is twice as long as the shortest leg while the longer leg is $\sqrt{3}$ times the length of the shorter leg.

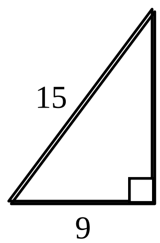
§6-5 PROBLEM SET

Find the length of the third side of each right triangle.

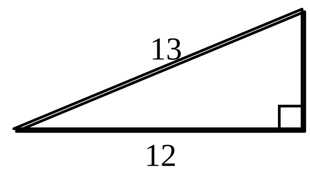
1.



2.

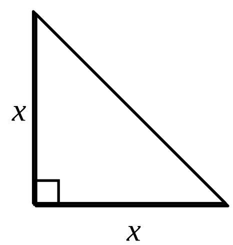


3.

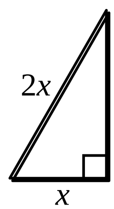


Find an expression for the length of the third side of each right triangle.

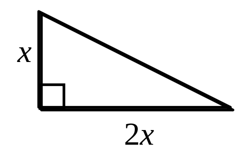
4.



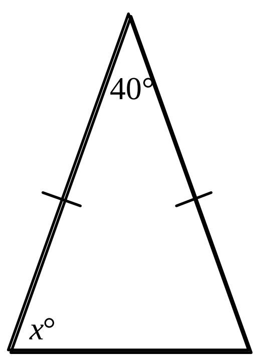
5.



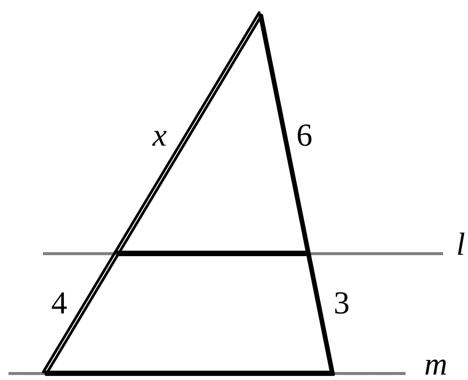
6.



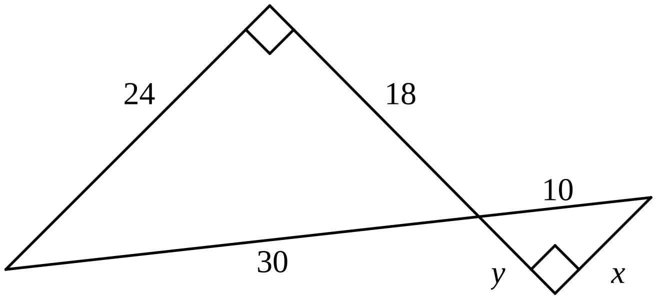
7. Find x



8. Lines l and m are parallel. Find x.



9. Solve for x and y.



§6-5 PROBLEM SOLUTIONS

1.	5	**2**.	12	**3**.	5	**4**.	$x\sqrt{2}$
5.	$x\sqrt{3}$	**6**.	$x\sqrt{5}$	**7**.	70°	**8**.	8

9. $x = 8$ and $y = 6$