§6-6 RIGHT TRIANGLE TRIGONOMETRY

In this section we consider the trigonometry of right triangles. The trigonometric functions of concern are the *sine*, *cosine* and *tangent* functions.

Definition

For a right triangle, the **sine** of an angle θ is the ratio of the length of the leg opposite θ to the length of the hypotenuse: $\sin\theta = \dfrac{\textit{opposite leg}}{\textit{hypotenuse}}$.

Definition

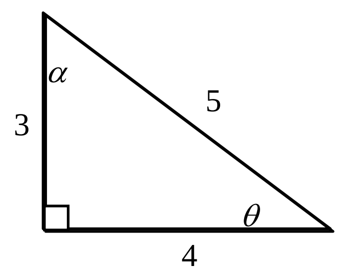
For a right triangle, the **cosine** of an angle θ is the ratio of the length of the leg adjacent θ to the length of the hypotenuse: $\cos\theta = \dfrac{\textit{adjacent leg}}{\textit{hypotenuse}}$.

Definition

For a right triangle, the **tangent** of an angle θ is the ratio of the length of the leg opposite θ to the length of the leg adjacent θ: $\tan\theta = \dfrac{\textit{opposite leg}}{\textit{adjacent leg}}$.

The mnemonic SOH CAH TOA is often used to recall the relationship of the sides of the right triangle to the trigonometric functions.

Example 1 Find the sine, cosine and tangent of θ and α in the right triangle shown below.



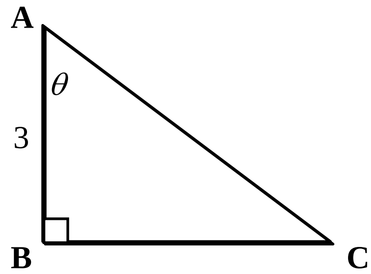
Solution For θ the opposite leg has length 3 and the adjacent leg has length 4. Therefore...

$$\sin\theta = \frac{\textit{opposite leg}}{\textit{hypotenuse}} = \frac{3}{5} \qquad \cos\theta = \frac{\textit{adjacent leg}}{\textit{hypotenuse}} = \frac{4}{5} \qquad \tan\theta = \frac{\textit{opposite leg}}{\textit{adjacent leg}} = \frac{3}{4}$$

For α the opposite leg has length 4 and the adjacent leg has length 3. Therefore...

$$\sin\alpha = \frac{\textit{opposite leg}}{\textit{hypotenuse}} = \frac{4}{5} \qquad \cos\alpha = \frac{\textit{adjacent leg}}{\textit{hypotenuse}} = \frac{3}{5} \qquad \tan\alpha = \frac{\textit{opposite leg}}{\textit{adjacent leg}} = \frac{4}{3}$$

Example 2 If $\cos\theta = x$ for the triangle below, then find the length of **AC**.



Solution

$$\cos\theta = \frac{\textit{adjacent leg}}{\textit{hypotenuse}}$$
$$x = \frac{3}{\mathbf{AC}}$$
$$\mathbf{AC}x = 3$$
$$\mathbf{AC} = \frac{3}{x}$$

Identity For any angle θ, $\sin^2\theta + \cos^2\theta = 1$.

Example 3 If $\sin\theta = \frac{1}{2}$ then find the cosine of θ.

Solution

$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \sin^2\theta$$
$$\cos^2\theta = 1 - \left(\frac{1}{2}\right)^2$$
$$\cos^2\theta = \frac{3}{4}$$
$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$

Identity For any angle θ, $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

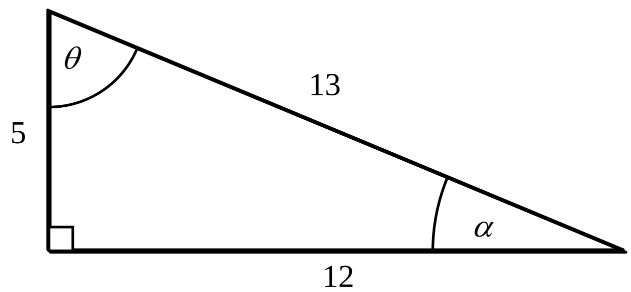
Example 4 If $\cos\theta = \frac{5}{13}$ and $\tan\theta = \frac{12}{5}$ then find the sine of θ.

Solution

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\tan\theta \cdot \cos\theta = \sin\theta$$
$$\frac{12}{5} \cdot \frac{5}{13} = \sin\theta$$
$$\frac{12}{13} = \sin\theta$$

§6-6 PROBLEM SET

Find each quantity for the right triangle below.



1. sin α

2. cos α

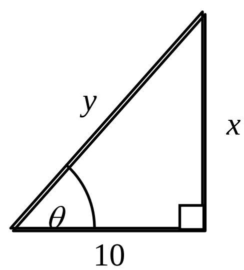
3. tan α

4. sin θ

5. cos θ

6. tan θ

Find the lengths of the missing sides of the right triangle below.



$\cos\theta = \frac{2}{3}$

$\tan\theta = \frac{\sqrt{5}}{2}$

7. Find x.

8. Find y.

Use the given quantities to calculate the missing quantity.

9. If $\sin\theta = \frac{1}{2}$ and $\cos\theta = \frac{\sqrt{3}}{2}$ then find tan θ.

10. If $\sin\theta = \frac{\sqrt{2}}{2}$ and $\tan\theta = 1$ then find cos θ.

11. If $\cos\theta = \frac{1}{3}$ and $\tan\theta = 2\sqrt{2}$ then find $\sin\theta$.

12. If $\cos\theta = \frac{\sqrt{5}}{3}$ and $\sin\theta > 0$ then find $\sin\theta$.

§6-6 PROBLEM SOLUTIONS

1.	$\frac{5}{13}$	**2**.	$\frac{12}{13}$	**3**.	$\frac{5}{12}$	**4**.	$\frac{12}{13}$
5.	$\frac{5}{13}$	**6**.	$\frac{12}{5}$	**7**.	15	**8**.	$5\sqrt{5}$
9.	$\frac{\sqrt{3}}{3}$	**10**.	$\frac{\sqrt{2}}{2}$	**11**.	$\frac{2\sqrt{2}}{3}$	**12**.	$\frac{2}{3}$