

§7-2**PROBABILITY****Definition**

Suppose an experiment has n possible outcomes and each outcome is equally likely to occur. If some event A were satisfied by m of those n , then **the probability that event A occurs** is $\frac{m}{n}$. This is written as $P(A) = \frac{m}{n}$.

Example 1

Suppose a small urn contains seven marbles, three of which are red and four of which are yellow. Suppose a single marble is drawn at random from the bag. What is the probability that a red marble is drawn?

Solution

There are 7 possible outcomes since there are seven marbles to choose from. Choosing any one of the red marbles will indicate a success. Since there are three red marbles in the urn, the probability that a red marble is drawn equals $\frac{3}{7}$.

$$P(\text{red marble is drawn}) = \frac{3}{7}.$$

Example 2

Suppose a fair, six-sided die is rolled. What is the probability that you roll a three?

Solution

There are six possible outcomes: $\{1, 2, 3, 4, 5, 6\}$, all of which are equally likely to occur. Rolling a 3 will indicate a success. Since there is only one way to roll a 3, the probability of rolling a three is $\frac{1}{6}$. $P(\text{red marble is drawn}) = \frac{1}{6}$.

Definition

Two events A and B are said to be **independent** of one another if the occurrence of event A does not effect the probability that event B occurs and vice-versa. In the case where the events are defined to be independent, the probability that both event A and event B occur, $P(AB)$, is the product of $P(A)$ and $P(B)$.

$$P(AB) = P(A) \cdot P(B)$$

Example 3

Using the urn in *Example 1* we will draw one marble, note its color, return it to the urn, shake up the urn and draw another marble. What is the probability that we will select two red marbles?

Solution

Let event A be the selection of a red marble on the first draw. Let event B be the selection of a red marble on the second draw. Since the marble was been replaced in the urn after the first draw, the total number of possible outcomes for the second draw was unchanged. In this case the two events are classified as independent events.

From *Example 1* we know that:

$$P(\text{red marble is drawn}) = \frac{3}{7} \text{ and so } P(A) = \frac{3}{7} \text{ and } P(B) = \frac{3}{7}.$$

$$\text{Therefore } P(AB) = P(A) \cdot P(B) = \frac{3}{7} \cdot \frac{3}{7} = \frac{9}{49}.$$

Definition

If the events A and B are not independent they are said to be **dependent** events.

Example 4

If we have already drawn a red marble from the urn in *Example 1* and we draw another marble without replacing the first, what is the probability that the second marble is also red?

Solution

If we have drawn a red marble then there are 2 red marbles and 4 white marbles left.

Therefore the probability of drawing a second red marble is

$$P(\text{drawing a second red marble}) = \frac{2}{6} = \frac{1}{3}.$$

Since the first marble was not replaced, the chances of choosing a red ball on the second draw is not the same as it was in the *Example 3*. Therefore the two events are said to be dependent.

Example 5

Suppose a fair coin is tossed in the air three separate times. On the first toss the coin lands tails. On the second toss the coin lands heads. What is the probability that on the third toss the coin lands tails?

Solution

On the third toss the number of possible outcomes is two, either heads or tails. It does not matter how the coin landed on the previous two tosses, these previous events have no affect on the outcome of the third toss. Therefore the probability of the coin landing tails on the third toss is:

$$P(\text{tails on third toss}) = \frac{1}{2}.$$

Example 6

Suppose we have drawn two kings from a regular deck of playing cards and not replaced them. What is the probability of drawing a third king on our next draw?

Solution

There were originally four kings out of fifty-two cards. Now there are only two kings left out of fifty cards. Thus

$$P(\text{king on third draw}) = \frac{2}{50} = \frac{1}{25}.$$

Example 7

Suppose that the integers from 2 through 15 inclusive are written on slips of paper which are to be randomly drawn from a hat. A 4 is randomly selected on the first draw and not replaced. What is the probability that an odd number is randomly selected on the second draw?

Solution

Before the first number is drawn there are 7 odd numbers out of 14 numbers total:

$$\{ 3, 5, 7, 9, 11, 13, 15 \}$$

After the 4 is drawn, there are still 7 odd numbers, but now there are 13 numbers total. Thus

$$P(\text{odd number on second draw}) = \frac{7}{13}.$$

One card is selected at random from a regular deck of 52 cards.

Find the probability of drawing each card described below.

1. A club
2. An ace
3. The ace of clubs
4. An ace or a club
5. An ace or a queen
6. Not the ace of clubs

A fair coin is flipped three times in row. Find the probability of each event below.

7. All three tosses come up heads.
8. At least two tosses come up heads.
9. At least one toss comes up heads.
10. Exactly one toss comes up heads.

A number, n , between 1 and 30 (inclusive) is selected at random.

Find each probability below.

11. $P(n \text{ is even})$
12. $P(n \leq 4)$
13. $P(n > 10)$
14. $P(n \text{ is divisible by } 3)$
15. $P(n \text{ is odd and has only one digit})$

A card is drawn from a 52 card bridge deck, replaced, the deck shuffled, and a second card is drawn. Find the probability of each event below.

16. Both cards are sevens.
17. Both cards are red.
18. Neither card is a heart.
19. Both cards are hearts.
20. The first card is a heart and the second card is a club.

Find the probability of each event below.

21. What is the probability that two cards drawn randomly without replacement from a regular deck of cards will both be kings?
22. Three dice are rolled. What is the probability that each produces a 1 or a 2?
23. A jar contains 3 blue marbles, 4 red marbles and 2 white marbles. A blue marble is selected at random and not replaced. What is the probability that the next randomly selected marble is also blue?
24. Two dice are tossed. What is the probability that their product is 12?
25. A lottery selects six numbers without replacement from the numbers 1 through 50. What is the probability that you will win the lottery with a single ticket?

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|-----|-----------------|-----|----------------|-----|----------------|-----|----------------|-----|------------------------|
| 1. | $\frac{1}{4}$ | 2. | $\frac{1}{13}$ | 3. | $\frac{1}{52}$ | 4. | $\frac{4}{13}$ | 5. | $\frac{2}{13}$ |
| 6. | $\frac{51}{52}$ | 7. | $\frac{1}{8}$ | 8. | $\frac{1}{2}$ | 9. | $\frac{7}{8}$ | 10. | $\frac{3}{8}$ |
| 11. | $\frac{1}{2}$ | 12. | $\frac{2}{15}$ | 13. | $\frac{2}{3}$ | 14. | $\frac{1}{3}$ | 15. | $\frac{1}{6}$ |
| 16. | $\frac{1}{169}$ | 17. | $\frac{1}{4}$ | 18. | $\frac{9}{16}$ | 19. | $\frac{1}{16}$ | 20. | $\frac{1}{16}$ |
| 21. | $\frac{1}{221}$ | 22. | $\frac{1}{27}$ | 23. | $\frac{1}{4}$ | 24. | $\frac{1}{9}$ | 25. | $\frac{1}{15,890,700}$ |