

§5-3**SYSTEMS OF LINEAR RELATIONS****Procedure****Solving Two Linear Equations by Substitution**

- Step 1: Choose either variable and solve one of the equations for that variable.
 Step 2: Substitute the result into the other equation.
 This will produce an equation with one variable.
 Step 3: Solve the equation for the remaining variable.
 Step 4: To find the other variable, substitute the result from *Step 3* into the equation developed in *Step 1*.
 Step 5: Write the solution as an ordered pair and check the answer.

Example 1

Solve the system of linear equations $\begin{cases} 2x + 3y = 8 \\ 4x + 3y = 4 \end{cases}$ by the substitution method.

Solution

Step 1: Choosing the first equation, solve for x .

$$\begin{aligned} 2x + 3y &= 8 \\ 2x &= 8 - 3y \\ x &= \frac{8 - 3y}{2} \end{aligned}$$

Step 2: Now substitute this expression for x into the second equation.

$$\begin{aligned} 4x + 3y &= 4 \\ 4\left(\frac{8 - 3y}{2}\right) + 3y &= 4 \end{aligned}$$

Step 3: Now solve this single variable equation for y .

$$\begin{aligned} 2(8 - 3y) + 3y &= 4 \\ 16 - 6y + 3y &= 4 \\ 16 - 3y &= 4 \\ -3y &= -12 \\ y &= 4 \end{aligned}$$

Step 4: Finally substitute the result $y = 4$ into $x = \frac{8 - 3y}{2}$ from *Step 1*.

$$x = \frac{8 - 3y}{2} = \frac{8 - 3(4)}{2} = \frac{8 - 12}{2} = -\frac{4}{2} = -2$$

Step 5: The solution to the system is $(-2, 4)$. Check this solution in both equations.

$$\begin{array}{ll} 2x + 3y \stackrel{?}{=} 8 & 4x + 3y \stackrel{?}{=} 4 \\ 2(-2) + 3(4) \stackrel{?}{=} 8 & 4(-2) + 3(4) \stackrel{?}{=} 4 \\ -4 + 12 \stackrel{?}{=} 8 & -8 + 12 \stackrel{?}{=} 4 \\ 8 \neq 8\checkmark & 4 = 4\checkmark \end{array}$$

Procedure**Solving Two Linear Equations by Elimination**

- Step 1: Write both equations in standard form: $Ax + By = C$.
- Step 2: Multiply one or both of the equations by appropriate numbers so that the coefficients of either x or y are the same.
- Step 3: Add or subtract the two equations to produce an equation with only one variable.
- Step 4: Solve the equation for the remaining variable.
- Step 5: To find the other variable, substitute the result from *Step 4* into either of the two original equations.
- Step 6: Write the solution as an ordered pair and check the answer.

Example 2

Solve the system of linear equations $\begin{cases} 2x - y - 12 = 0 \\ 3x + 2y + 3 = 0 \end{cases}$ by the elimination method.

Solution

Step 1: Write both equations in standard form: $\begin{cases} 2x - y = 12 \\ 3x + 2y = -3 \end{cases}$

Step 2: Multiply the first equation by 2 to make the coefficients of y equal.

$$\begin{aligned} 2x - y &= 12 \\ 2(2x - y) &= 2(12) \\ 4x - 2y &= 24 \end{aligned}$$

Step 3: Next, add the two equations together to eliminate the y variable.

$$\begin{array}{r} 4x - 2y = 24 \\ + \quad 3x + 2y = -3 \\ \hline 7x + 0y = 21 \end{array}$$

Step 4: Now solve this single variable equation for x .

$$\begin{aligned} 7x &= 21 \\ x &= 3 \end{aligned}$$

Step 5: Finally, substitute the result $x = 3$ into $3x + 2y + 3 = 0$ we can solve for y .

$$\begin{aligned} 3x + 2y + 3 &= 0 \\ 3(3) + 2y + 3 &= 0 \\ 2y + 12 &= 0 \\ 2y &= -12 \\ y &= -6 \end{aligned}$$

Step 6: The solution to the system is $(3, -6)$. Check this solution in both equations.

$$\begin{array}{rcl} 2x - y - 12 & ? & 0 \\ 2(3) - (-6) - 12 & ? & 0 \\ 6 + 6 - 12 & ? & 0 \\ 0 & = & 0\checkmark \end{array} \qquad \begin{array}{rcl} 3x + 2y + 3 & ? & 0 \\ 3(3) + 2(-6) + 3 & ? & 0 \\ 9 - 12 + 3 & ? & 0 \\ 0 & = & 0\checkmark \end{array}$$

Solve each system of equations using substitution.

$$1. \begin{cases} 3x + 2y = 12 \\ 2x - 1 = y \end{cases}$$

$$2. \begin{cases} 9x + 5y = -28 \\ x - 3y = 4 \end{cases}$$

$$3. \begin{cases} 5x + 3y = 29 \\ 7 - x = y \end{cases}$$

$$4. \begin{cases} 3x - 5y = 1 \\ 2x - y = 3 \end{cases}$$

$$5. \begin{cases} x = 2y - 11 \\ 19 = 3x + 7y \end{cases}$$

$$6. \begin{cases} y - 3x = -2 \\ 2x + 5y = 7 \end{cases}$$

$$7. \begin{cases} 5x - 8y = -1 \\ 5x - 9 = 3y \end{cases}$$

$$8. \begin{cases} 2x + 3y = 1 \\ 3x + 4y = 2 \end{cases}$$

$$9. \begin{cases} y = 2x + 3 \\ 2 = 3y + x \end{cases}$$

Solve each system of equations using elimination.

$$10. \begin{cases} x + y = 11 \\ x - y = 7 \end{cases}$$

$$11. \begin{cases} 2x - y = 5 \\ 3x + y = 25 \end{cases}$$

$$12. \begin{cases} x + 2y = 12 \\ x - y = 4 \end{cases}$$

$$13. \begin{cases} 2x + 4 = y \\ 5x - y = 2 \end{cases}$$

$$14. \begin{cases} 3x + 2y = 14 \\ x - y = 3 \end{cases}$$

$$15. \begin{cases} 2x + 7y = 4 \\ x + 11 = 3y \end{cases}$$

$$16. \begin{cases} 5x + 2y = 40 \\ 2x + y = 14 \end{cases}$$

$$17. \begin{cases} 4x = y + 9 \\ 8 = 3x - y \end{cases}$$

$$18. \begin{cases} 6x = y + 5 \\ y = 9x + 4 \end{cases}$$

Solve each system of equations using any method.

$$19. \begin{cases} 2x - y = 7 \\ 2x - 3y = 9 \end{cases}$$

$$20. \begin{cases} 2x - y = 8 \\ 3x + y = 17 \end{cases}$$

$$21. \begin{cases} x - y = 8 \\ 2x - 3y = 21 \end{cases}$$

$$22. \begin{cases} 4x - 3y = 5 \\ 10x + 2y = 3 \end{cases}$$

$$23. \begin{cases} 2x + 9y = 2 \\ 3x - 6y = -10 \end{cases}$$

$$24. \begin{cases} x - 2y = 2 \\ 15x + 10y = 2 \end{cases}$$

$$25. \begin{cases} 2x - 3y = 7 \\ 4x + 2y = 30 \end{cases}$$

$$26. \begin{cases} 0.4x - 0.3y = 3.4 \\ 0.6x + 0.9y = 2.4 \end{cases}$$

$$27. \begin{cases} 2x + 1.5y = 10.5 \\ 0.8x - 0.5y = 0.9 \end{cases}$$

$$28. \begin{cases} \frac{1}{2}y = \frac{3}{2}x - 2 \\ \frac{1}{2} = x + \frac{1}{2}y \end{cases}$$

$$29. \begin{cases} \frac{x}{2} + \frac{y}{3} = 10 \\ \frac{x}{3} + \frac{y}{2} = 10 \end{cases}$$

$$30. \begin{cases} \frac{7x}{2} - \frac{5y}{4} = -12 \\ \frac{3x}{2} + 4 = \frac{y}{4} \end{cases}$$

$$31. \begin{cases} \frac{1}{3}x - \frac{1}{3}y = -1 \\ y + \frac{1}{2}x = 0 \end{cases}$$

$$32. \begin{cases} \frac{3x}{4} + \frac{5y}{2} = 8 \\ \frac{3x}{2} - 5 = \frac{y}{2} \end{cases}$$

$$33. \begin{cases} \frac{x+1}{3} - \frac{y}{4} = -1 \\ \frac{y}{3} - \frac{3x+1}{4} = 0 \end{cases}$$

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|---|---|---|--|
| 1. (2, 3) | 2. (-2, -2) | 3. (4, 3) | 4. (2, 1) |
| 5. (-3, 4) | 6. (1, 1) | 7. (3, 2) | 8. (2, -1) |
| 9. (-1, 1) | 10. (9, 2) | 11. (6, 7) | 12. $\left(\frac{20}{3}, \frac{8}{3}\right)$ |
| 13. (2, 8) | 14. (4, 1) | 15. (-5, 2) | 16. (12, -10) |
| 17. (1, -5) | 18. (-3, -23) | 19. (3, -1) | 20. (5, 2) |
| 21. (3, -5) | 22. $\left(\frac{1}{2}, -1\right)$ | 23. $\left(-2, \frac{2}{3}\right)$ | 24. $\left(\frac{3}{5}, -\frac{7}{10}\right)$ |
| 25. $\left(\frac{13}{2}, 2\right)$ | 26. (7, -2) | 27. (3, 3) | 28. (1, -1) |
| 29. (12, 12) | 30. (-2, 4) | 31. (-2, 1) | 32. (4, 2) |
| 33. (5, 12) | | | |