

§2-2**POLYNOMIAL EXPRESSIONS****Definition**

Terms with exactly the same variable parts (the same variables raised to the same powers) are called **like terms**. Terms with different variable parts are called **unlike terms**.

Example 1

Simplify the polynomial expression $7x^2 + 3xy - 4x^2 + xy - y^2$ by combining like terms.

Solution

$$\begin{aligned} 7x^2 + 3xy - 4x^2 + xy - y^2 &= 7x^2 - 4x^2 + 3xy + xy - y^2 \\ &= (7 - 4)x^2 + (3 + 1)xy - y^2 \\ &= 3x^2 + 4xy - y^2 \end{aligned}$$

Example 2

Given $x = -2$ and $y = 3$, what is the value of $xy^2 - 2xy$.

Solution

$$\begin{aligned} xy^2 - 2xy &= (-2)(3)^2 - 2(-2)(3) \\ &= (-2)(9) - 2(-6) \\ &= -18 + 12 \\ &= -6 \end{aligned}$$

Procedure**Multiplying Polynomials**

Two polynomials are multiplied together by multiplying each term of the first polynomial by each term of the second polynomial and then summing the products.

Example 3

Find the product of $(x^2 - y)(2x + y^2 - 3)$.

Solution

$$\begin{aligned} (x^2 - y)(2x + y^2 - 3) &= (x^2)(2x) + (x^2)(y^2) - (x^2)(3) - (y)(2x) - (y)(y^2) + (y)(3) \\ &= 2x^3 + x^2y^2 - 3x^2 - 2xy - y^3 + 3y \end{aligned}$$

Formulas**Description**

Difference of two squares

Perfect Square Trinomials

Sum of two cubes

Difference of two cubes

Formula

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example 4

Factor $9x^2 - 64y^2$.

Solution

Since the expression $9x^2 - 64y^2$ is the difference of two squares we have

$$\begin{aligned} 9x^2 - 64y^2 &= (3x)^2 - (8y)^2 \\ &= (3x - 8y)(3x + 8y) \end{aligned}$$

Example 5

Factor $16m^2 - 16mn^2 + 4n^4$.

Solution

$$\begin{aligned} 16m^2 - 16mn^2 + 4n^4 &= 4[4m^2 - 4mn^2 + n^4] \\ &= 4[(2m)^2 - 2 \cdot (2m)(n^2) + (n^2)^2] \\ &= 4(2m - n^2)^2 \end{aligned}$$

Evaluate each polynomial for the given values.

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|---|--|
| 1. $3x^2 + 2$ where $x = 1$ | 2. $-x^2$ where $x = -2$ |
| 3. $2x^2 + 3x - 3$ where $x = -2$ | 4. $4t^2 - 6t + 2$ where $t = \frac{1}{2}$ |
| 5. $x^2y - 2x + 1$ where $x = -1$ and $y = 3$ | 6. $3ab - 2b^2 + 2$ where $a = 2$ and $b = -2$ |

Simplify by combining like terms.

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|---|---|
| 7. $(7x - 2) + (5x + 22)$ | 8. $(3y + 4) - (4y - 12)$ |
| 9. $(3x^2 - 2x + 7) + (x^2 + 4x - 2)$ | 10. $(5x^2 + 3x - 1) - (2x^2 - 4x - 2)$ |
| 11. $(2a^2b + 5ab^2) + (ab^2 + 3ab - a^2b)$ | 12. $(5b^3 - ab + 4a^2) - (a^2 - b - ab)$ |
| 13. $(3x^2 - 2x + 5) + (-3x^2 + 2x + 1)$ | 14. $(xy^2 + y - 2) - (x^2y + 2x + y)$ |

Multiply and then simplify.

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|-----------------------------|------------------------------------|
| 15. $4x^2(2x - 9)$ | 16. $4y^2(3y^2 - 5y + 7)$ |
| 17. $4a^2(5b^2 - 3b + 4)$ | 18. $(2x + 3)(x - 1)$ |
| 19. $(x - 2)(x^2 + 2x - 1)$ | 20. $3(t - 6)(t^2 - t + 3)$ |
| 21. $(x - 5)(x + 1)^2$ | 22. $(2r^2 - r + 7)(r^2 + 2r - 3)$ |

Factor completely.

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|-----------------------------|--------------------------------|
| 23. $x^2 - 9$ | 24. $64r^2 - 25s^2$ |
| 25. $4x^2 + 16$ | 26. $98x^3 - 50x$ |
| 27. $9x^2 - 60x + 100$ | 28. $25x^2 + 40x + 16$ |
| 29. $27x^4 + 72x^3 + 48x^2$ | 30. $45a^3 + 60a^2 + 20a$ |
| 31. $64x^2 - 80xy + 25y^2$ | 32. $16a^4 - 72a^2b^2 + 81b^4$ |
| 33. $x^2 + 4$ | 34. $x^3 + 8$ |
| 35. $x^3 - 27$ | 36. $5x^5 + 40x^2$ |
| 37. $4t^4 - 32t$ | 38. $3y^3 + 27y$ |
| 39. $64r^3 + 27s^3$ | 40. $a^3 + 8b^6$ |

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|--------------------------------------|--------------------------------------|
| 1. 5 | 2. -4 |
| 3. -1 | 4. 0 |
| 5. 6 | 6. -18 |
| 7. $12x + 20$ | 8. $-y + 16$ |
| 9. $4x^2 + 2x + 5$ | 10. $3x^2 + 7x + 1$ |
| 11. $a^2b + 3ab + 6ab^2$ | 12. $5b^3 + 3a^2 + b$ |
| 13. 6 | 14. $xy^2 - x^2y - 2x - 2$ |
| 15. $8x^3 - 36x^2$ | 16. $12y^4 - 20y^3 + 28y^2$ |
| 17. $20a^2b^2 - 12a^2b + 16a^2$ | 18. $2x^2 + x - 3$ |
| 19. $x^3 - 5x + 2$ | 20. $3t^3 - 21t^2 + 27t - 54$ |
| 21. $x^3 - 3x^2 - 9x - 5$ | 22. $2r^4 + 3r^3 - r^2 + 17r - 21$ |
| 23. $(x - 3)(x + 3)$ | 24. $(8r - 5s)(8r + 5s)$ |
| 25. $4(x^2 + 4)$ | 26. $2x(7x + 5)(7x - 5)$ |
| 27. $(3x - 10)^2$ | 28. $(5x + 4)^2$ |
| 29. $3x^2(3x + 4)^2$ | 30. $5a(3a + 2)^2$ |
| 31. $(8x - 5y)^2$ | 32. $(2a + 3b)^2(2a - 3b)^2$ |
| 33. $x^2 + 4$ | 34. $(x + 2)(x^2 - 2x + 4)$ |
| 35. $(x - 3)(x^2 + 3x + 9)$ | 36. $5x^2(x + 2)(x^2 - 2x + 4)$ |
| 37. $4t(t - 2)(t^2 + 2t + 4)$ | 38. $3y(y^2 + 9)$ |
| 39. $(4r + 3s)(16r^2 - 12rs + 9s^2)$ | 40. $(a + 2b^2)(a^2 - 2ab^2 + 4b^4)$ |